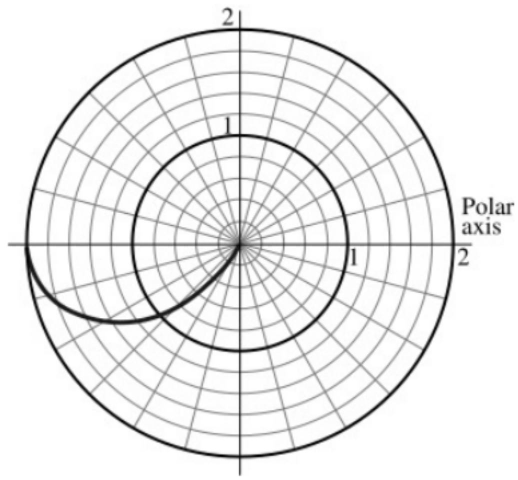
Spring 2025 Laird Homework 8

1. The figure below shows a portion of the graph of the polar function $r = 2 - 4\cos\theta$ in the polar coordinate system for $a \leq \theta \leq b$.



(a) If $0 \leq a < b \leq 2\pi$, find the values of a and b that represent the portion of the curve shown.

2. Consider the polar function $r = 3 \cdot (\cos\theta)$.

(a) Determine which quadrants contain points of this curve.

(b) Consider the function $r = -3 \cdot (\cos\theta)$. Determine which quadrants contain points of *this* curve.

(c) Consider the function $r = 3 \cdot (\cos\theta) + 4$. Determine which quadrants contain points of *this* curve.

3. Express the complex number $7 + 7i$ in polar form $r(\cos\theta + i\sin\theta)$.

4. Consider the polar function $r = \frac{\theta}{2}$ defined for $\theta \geq 0$.

(a) Explain how the radius changes as θ increases.

(b) Describe the shape of the function.

5. Consider the polar function $r = 2 - \cos(-\theta)$.

(a) As θ changes from $\frac{\pi}{2}$ to π, describe how the distance between the origin and the point with polar coordinates $(f(\theta), \theta)$ changes.

(b) Find the minimum value of r in the interval $\frac{\pi}{2} \leq \theta \leq \pi$.

(c) Find the maximum value of r in the interval $\frac{\pi}{2} \leq \theta \leq \pi$.

6. Consider the polar function $f(\theta) = r = 1 + 2\sin\theta$ for $0 \leq \theta \leq 2\pi$.

(a) Find all intervals where r is decreasing.

(b) Find all intervals where the distance between the point with polar coordinates $(f(\theta), \theta)$ and the origin is increasing.

NB: These two questions are related, but not identical.

7. Consider the polar function $r = -1 + \sin\theta$ for $0 \leq \theta \leq 2\pi$.

(a) Find the minimum value of r in the interval $0 < \theta < \frac{\pi}{2}$.

(b) Find all intervals where the points on the graph lie above or on the x-axis.

8. Consider the polar function $r = 2\sin(2\theta)$ for $0 \leq \theta \leq \pi$.

(a) Find all values of θ in this interval where r has an extremum.