Solutions

Question 6 Solutions:

A) To find $\cos(5\pi/12)$, we use the fact that $5\pi/12 = \pi/4 + \pi/6$ and apply the cosine addition formula:

$$\begin{aligned}
\cos(a+b) &= \cos(a)\cos(b) - \sin(a)\sin(b) \\
\cos\left(\frac{5\pi}{12}\right) &= \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right) \\
&= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} \\
&= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \\
&= \frac{\sqrt{6}-\sqrt{2}}{4}
\end{aligned}$$

B) For $\sin(5\pi/12)$, we use the sine addition formula:

$$\begin{aligned}
\sin(a+b) &= \sin(a)\cos(b) + \cos(a)\sin(b) \\
\sin\left(\frac{5\pi}{12}\right) &= \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right) \\
&= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} \\
&= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} \\
&= \frac{\sqrt{6}+\sqrt{2}}{4}
\end{aligned}$$

C) For $\sin(7\pi/12)$, note that $\frac{7\pi}{12} = \frac{\pi}{4} + \frac{\pi}{3}$, so:

$$\begin{aligned}
\sin\left(\frac{7\pi}{12}\right) &= \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) + \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right) \\
&= \frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} \\
&= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} \\
&= \frac{\sqrt{2}+\sqrt{6}}{4}
\end{aligned}$$

D) Using the cosine addition formula:

$$\begin{aligned}
\cos\left(x+\frac{\pi}{4}\right) &= \cos(x)\cos\left(\frac{\pi}{4}\right) - \sin(x)\sin\left(\frac{\pi}{4}\right) \\
&= \cos(x)\cdot\frac{\sqrt{2}}{2} - \sin(x)\cdot\frac{\sqrt{2}}{2} \\
&= \frac{\sqrt{2}}{2}(\cos(x) - \sin(x))
\end{aligned}$$

E) Using the sine addition formula:

$$\begin{aligned}
\sin(\pi+x) &= \sin(\pi)\cos(x) + \cos(\pi)\sin(x) \\
&= 0\cdot\cos(x) + (-1)\cdot\sin(x) \\
&= -\sin(x)
\end{aligned}$$

F) Using the cosine subtraction formula:

$$\begin{aligned}\cos(x-\pi) &= \cos(x)\cos(\pi)+\sin(x)\sin(\pi)\\ &= \cos(x)\cdot(-1)+\sin(x)\cdot 0\\ &= -\cos(x)\end{aligned}$$

Question 7 Solutions:

A) The frequency of a function is the reciprocal of the period. From the graph, we can see that $f(x)$ completes one full cycle from $x=-\pi$ to $x=3\pi$. Therefore:

$$\begin{aligned}\text{Period} &= 4\pi\\ \text{Frequency} &= \frac{1}{\text{Period}}=\frac{1}{4\pi}\end{aligned}$$

B) To find $f(1000\pi)$, we need to determine where in the cycle this input falls. Since the period is 4π, we can use modular arithmetic:

$$\begin{aligned}1000\pi &= 250\cdot 4\pi+0\\ \text{So } f(1000\pi) &= f(0)=0\end{aligned}$$

We can verify from the graph that $f(0)=0$.

C) Similarly for $f(1001\pi)$:

$$\begin{aligned}1001\pi &= 250\cdot 4\pi+\pi\\ \text{So } f(1001\pi) &= f(\pi)=1\end{aligned}$$

We can verify from the graph that $f(\pi)=1$.

D) To determine the slope of $f(x)$ at $x=10\pi$, we first find where in the cycle this input falls:

$$\begin{aligned}10\pi &= 2\cdot 4\pi+2\pi\\ \text{So } f(10\pi) &= f(2\pi)\end{aligned}$$

Looking at the graph, at $x=2\pi$ (and equivalently at $x=10\pi$), the function is decreasing. Therefore, the slope is negative.

E) To find $g(10\pi)$, we use the fact that $g(x)$ has period 2π and find where in the cycle this input falls:

$$\begin{aligned}10\pi &= 5\cdot 2\pi+0\\ \text{So } g(10\pi) &= g(0)=2\end{aligned}$$

F) For $g(-7\pi/2)$, we need to first make the input positive by adding periods:

$$\begin{aligned}-\frac{7\pi}{2} &= -\frac{7\pi}{2}+5\cdot 2\pi=\frac{-7\pi+10\pi}{2}=\frac{3\pi}{2}\\ \text{So } g\left(-\frac{7\pi}{2}\right) &= g\left(\frac{3\pi}{2}\right)=0\end{aligned}$$

From the table, we can verify that $g(3\pi/2)=0$.

Question 15 Solutions:

A) Consider angle α where $\sin(\alpha) = 0.1$. To find the possible values of $\cos(\alpha)$, we use the Pythagorean identity:

$$\begin{aligned} \sin^2(\alpha) + \cos^2(\alpha) &= 1 \\ \cos^2(\alpha) &= 1 - \sin^2(\alpha) \\ &= 1 - (0.1)^2 \\ &= 1 - 0.01 \\ &= 0.99 \end{aligned}$$

Therefore, $\cos(\alpha) = \pm\sqrt{0.99}$, giving us two possible values:

$$\begin{aligned} \cos(\alpha) &= \sqrt{0.99} \\ \text{or } \cos(\alpha) &= -\sqrt{0.99} \end{aligned}$$

B) To find $\sin(2x)$ in terms of $\sin(x)$ and $\cos(x)$, we use the double angle formula:

$$\begin{aligned} \sin(x + x) &= \sin(x)\cos(x) + \cos(x)\sin(x) \\ \sin(2x) &= 2\sin(x)\cos(x) \end{aligned}$$

C) For $\frac{\sin x}{\sec x} + \frac{\cos x}{\csc x} = \sin(a)$, we simplify the left side:

$$\begin{aligned} &= \frac{\sin x}{\frac{1}{\cos x}} + \frac{\cos x}{\frac{1}{\sin x}} \\ &= \sin x \cdot \cos x + \cos x \cdot \sin x \\ &= 2\sin x \cos x \end{aligned}$$

From part B, we know that $2\sin x \cos x = \sin(2x)$. Therefore:

$$\sin(a) = \sin(2x)$$

This means that $a = 2x$, so our answer is $a = 2x$.

D) For the integer value b such that $b = \sin^2(x) \cdot (1 + \cot^2(x))$, we simplify:

$$\begin{aligned} b &= \sin^2(x) \cdot \left(1 + \frac{\cos^2(x)}{\sin^2(x)}\right) \\ &= \sin^2(x) + \sin^2(x) \cdot \frac{\cos^2(x)}{\sin^2(x)} \\ &= \sin^2(x) + \cos^2(x) \\ &= 1 \end{aligned}$$

Therefore, $b = 1$.

Question 16 Solutions:

A) Based on the residual plots:

- Model 1 (Quadratic): Shows small, random scatter around zero
- Model 2 (Exponential): Shows clear curved pattern
- Model 3 (Linear): Shows clear curved pattern

Model 1 (quadratic regression) provides the best fit as its residual plot shows random scatter.

B) Using exponential regression: $P(t) = 3605.939(1.1193)^t$

C) Population prediction for 2027 ($t = 18$):

$$\begin{aligned} P(18) &= 3,605.939(1.1193)^{18} \\ &= 27,407 \text{ people} \end{aligned}$$

Question 17 Solutions:

Use your calculator to find the sinusoidal regression model: $y = 1.23372 \cdot \sin(0.519979x - 2.98293) + 2.91505$

A) For a function of the form $A \sin(Bx + C) + D$, the amplitude is $|A|$.

$$\text{Amplitude} = |1.23372| = 1.23372$$

B) The midline of a sinusoidal function is the horizontal line around which the function oscillates, which corresponds to the vertical shift D.

$$\text{Midline} : y = 2.91505$$

C) To calculate the residual for the point $(12, 2.5)$, we need to find the predicted value at $x = 12$ and subtract it from the actual value.

$$\begin{aligned} \text{Predicted value} &= 1.23372 \cdot \sin(0.519979 \cdot 12 - 2.98293) + 2.91505 \\ &= 1.23372 \cdot \sin(6.239748 - 2.98293) + 2.91505 \\ &= 1.23372 \cdot \sin(3.256818) + 2.91505 \\ &= 1.23372 \cdot (-0.114971) + 2.91505 \\ &\approx -0.141841 + 2.91505 \\ &\approx 2.7732 \end{aligned}$$

Now we can calculate the residual:

$$\begin{aligned} \text{Residual} &= \text{Actual value} - \text{Predicted value} \\ &= 2.5 - 2.7732 \\ &\approx -0.2732 \end{aligned}$$

D) Since the residual is negative (the actual value is less than the predicted value), our model overestimates the depth at $t = 12$ hours.