Spring 2025 Laird Homework 7 Solutions

Section 1

Part A

$(2, 2\sqrt{3})$

For point $(4, \frac{\pi}{3})$ in polar form:

$$x = r\cos(\theta) = 4\cos(\frac{\pi}{3}) = 4(\frac{1}{2}) = 2$$
$$y = r\sin(\theta) = 4\sin(\frac{\pi}{3}) = 4(\frac{\sqrt{3}}{2}) = 2\sqrt{3}$$

Part B

$(3\sqrt{2}, -\frac{\pi}{4})$

For point $(3, -3)$ in rectangular form:

$$r = \sqrt{x^2 + y^2} = \sqrt{3^2 + (-3)^2} = \sqrt{18} = 3\sqrt{2}$$
$$\theta = \arctan(\frac{y}{x}) = \arctan(-1) = -\frac{\pi}{4}$$

Because the point is in quadrant 4 and our angle $\frac{-\pi}{4}$ is in quadrant 4, we do not need to add or subtract π to get our angle.

Part C

$(3.9508, 0.6257)$

For point $(4, \frac{\pi}{20})$ in rectangular form:

$$x = r\cos(\theta) = 4\cos(\frac{\pi}{20}) = 4(0.9878) = 3.9508$$
$$y = r\sin(\theta) = 4\sin(\frac{\pi}{20}) = 4(0.1571) = 0.6257$$

Part D

$(\sqrt{82}, 1.4601)$

For point $(1, 9)$ in polar form:

$$r = \sqrt{x^2 + y^2} = \sqrt{1^2 + 9^2} = \sqrt{82} \approx 9.0554$$
$$\theta = \arctan(\frac{y}{x}) = \arctan(\frac{9}{1}) = 1.4601$$

Because the point is in quadrant 1, we do not need to adjust our angle.

Part E

$(\sqrt{82}, 4.6017)$

For point $(-1, -9)$ in polar form:

$$r = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + (-9)^2} = \sqrt{82}$$
$$\theta = \arctan(\frac{y}{x}) = \arctan(\frac{-9}{-1}) = \arctan(9) = 1.4601$$

Because the point is in quadrant 3, we need to add π to our angle: $1.4601 + \pi = 4.6017$

Section 2

Part A

$-8 + 3i$

A 90° counterclockwise rotation multiplies by i: $(3 + 8i)(i) = 3i + 8i^2 = 3i - 8 = -8 + 3i$

Part B

i

A 90° counterclockwise rotation is equivalent to multiplying by i

Part C

$\sqrt{5}(\cos(-1.1071) + i\sin(-1.1071))$

For $r = 1 - 2i$:

$$r = \sqrt{1^2 + (-2)^2} = \sqrt{5}$$
$$\theta = \arctan(\frac{-2}{1}) = -1.1071$$

Part D

$\sqrt{5}(\cos(2.0344) + i\sin(2.0344))$

A 180° rotation adds π to the angle from part C: $-1.1071 + \pi = 2.0344$

Part E

$4\sqrt{2}(\cos(\frac{13\pi}{12}) + i\sin(\frac{13\pi}{12}))$

For $p = -4 + 4i$, first convert to polar form:

$$r = \sqrt{(-4)^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$$
$$\theta = \arctan(\frac{4}{-4}) + \pi = -\frac{\pi}{4} + \pi = \frac{3\pi}{4}$$

Note that $\frac{13\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{3}$, so:

$$\begin{aligned}
\cos\left(\frac{13\pi}{12}\right) &= \cos\left(\frac{3\pi}{4} + \frac{\pi}{3}\right) \\
&= \cos\left(\frac{3\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) - \sin\left(\frac{3\pi}{4}\right)\sin\left(\frac{\pi}{3}\right) \\
&= -\frac{\sqrt{2}}{2}\cdot\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} \\
&= -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} \\
&= -\frac{\sqrt{2}+\sqrt{6}}{4}
\end{aligned}$$

$$\begin{aligned}
\sin\left(\frac{13\pi}{12}\right) &= \sin\left(\frac{3\pi}{4} + \frac{\pi}{3}\right) \\
&= \sin\left(\frac{3\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) + \cos\left(\frac{3\pi}{4}\right)\sin\left(\frac{\pi}{3}\right) \\
&= \frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \left(-\frac{\sqrt{2}}{2}\right)\cdot\frac{\sqrt{3}}{2} \\
&= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} \\
&= \frac{\sqrt{2}-\sqrt{6}}{4}
\end{aligned}$$

Therefore:

$$\begin{aligned}
p_{rotated} &= 4\sqrt{2}\left(\cos\left(\frac{13\pi}{12}\right) + i\sin\left(\frac{13\pi}{12}\right)\right) \\
&= 4\sqrt{2}\left(-\frac{\sqrt{2}+\sqrt{6}}{4} + i\frac{\sqrt{2}-\sqrt{6}}{4}\right) \\
&= -(2+\sqrt{12}) + (2-\sqrt{12})i \\
&= -(2+2\sqrt{3}) + (2-2\sqrt{3})i
\end{aligned}$$

Part F

$\cos(\frac{\pi}{3}) + i\sin(\frac{\pi}{3})$

Adding $\frac{\pi}{3}$ rotation: $\theta_{new} = \frac{3\pi}{4} + \frac{\pi}{3} = \frac{13\pi}{12}$

Section 3

Part A

$1(\cos(2\pi) + i\sin(2\pi)) = 1$ **OR** $1(\cos(0) + i\sin(0)) = 1$

For $m = i$:

- $r = 1$
- $\theta = \frac{\pi}{2}$
- $m^4 = (1(\cos(\frac{\pi}{2}) + i\sin(\frac{\pi}{2})))^4 = 1(\cos(2\pi) + i\sin(2\pi)) = 1$

Part B

$128\sqrt{2}(\cos(\frac{5\pi}{4}) + i\sin(\frac{5\pi}{4})) = -128 - 128i$

For $n = 2 + 2i$:

- $r = \sqrt{2^2 + 2^2} = 2\sqrt{2}$
- $\theta = \arctan(\frac{2}{2}) = \frac{\pi}{4}$
- $n^5 = (2\sqrt{2}(\cos(\frac{\pi}{4}) + i\sin(\frac{\pi}{4})))^5$
- $r_{new} = (2\sqrt{2})^5 = 128\sqrt{2}$
- $\theta_{new} = 5(\frac{\pi}{4}) = \frac{5\pi}{4}$

Part C

$2(\cos(\frac{\pi}{6} + \frac{2\pi k}{3}) + i\sin(\frac{\pi}{6} + \frac{2\pi k}{3}))$, where $k = 0, 1, 2$

For $w^3 = 8(\cos(\frac{\pi}{2}) + i\sin(\frac{\pi}{2}))$:

- $r = \sqrt[3]{8} = 2$
- $\theta = \frac{\pi}{6} + \frac{2\pi k}{3}$, where $k = 0, 1, 2$

Therefore the three solutions are:

$$w_1 = 2(\cos(\frac{\pi}{6}) + i\sin(\frac{\pi}{6})) = \sqrt{3} + i$$
$$w_2 = 2(\cos(\frac{5\pi}{6}) + i\sin(\frac{5\pi}{6})) = -\sqrt{3} + i$$
$$w_3 = 2(\cos(\frac{3\pi}{2}) + i\sin(\frac{3\pi}{2})) = -2i$$

Section 4

Yes, all three are complex numbers. A complex number is of the form $a + bi$, where a and b are real numbers. a and b can be any real numbers, including 0.