Quiz 6 Solutions

(2) Solutions:

A) For an equilateral triangle with height $h = 6$:
Let s be the side length. The height divides the base into two equal parts.
Using the 30-60-90 triangle formed: $6 = \frac{s\sqrt{3}}{2}$
Therefore: $s = \frac{12}{\sqrt{3}} = 4\sqrt{3}$ meters

B) For the 30-60-90 triangle with hypotenuse 5:
Side a (opposite to $\frac{\pi}{3}$ or $60°$) $= 5 \cdot \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{2}$ units

C) Side b (opposite to $\frac{\pi}{6}$ or $30°$) $= 5 \cdot \frac{1}{2} = \frac{5}{2}$ units

D) For point $(10, 10\sqrt{3})$ on circle:
$\tan(\theta) = \frac{10\sqrt{3}}{10} = \sqrt{3}$
Therefore, $\theta = \frac{\pi}{3}$ radians

E) Using the Pythagorean theorem:
$r^2 = 10^2 + (10\sqrt{3})^2 = 100 + 300 = 400$
Therefore, $r = 20$ units

(4) Solutions:

A) For point $(0.4, -0.9)$ on unit circle:
$\tan(\theta) = \frac{-0.9}{0.4} = -2.25$

B) At $\frac{\pi}{2}$ radians:
$y = r\sin(\frac{\pi}{2}) = 30 \cdot 1 = 30$ units

C) At $\frac{5\pi}{3}$ radians:
$y = r\sin(\frac{5\pi}{3}) = 20 \cdot \sin(\frac{5\pi}{3}) = 20 \cdot (-\frac{\sqrt{3}}{2}) = -10\sqrt{3}$ units

D) At $\frac{\pi}{3}$ radians:
Slope $= \tan(\frac{\pi}{3}) = \sqrt{3}$

E) Sine is positive in Quadrants I and II

F) Tangent is positive in Quadrants I and III

(5) Solutions:

A) Given $\cos(\theta) = 0.4$:
$x = r\cos(\theta) = 5 \cdot 0.4 = 2$ units

B) $\cos(\frac{7\pi}{4}) = \frac{\sqrt{2}}{2}$

C) $\tan(\frac{\pi}{3}) = \sqrt{3}$

D) On $[2\pi, 4\pi]$, $\tan(\frac{x}{2}) = 0$ when $\frac{x}{2} = \pi$ or 2π

Therefore, $x = 2\pi, 4\pi$

E) If $\tan(\alpha) = \frac{3}{8}$, then that is the slope of the terminal ray: $\frac{3}{8}$

(13) Solutions:

A) For $n(x) = 2\arccos(x)$:

Let's analyze the function:

- $\arccos(x)$ has range $[0, \pi]$ and domain $[-1, 1]$
- When we multiply by 2, the range doubles to $[0, 2\pi]$
- The domain remains $[-1, 1]$

Looking at the graphs:

- Graph A ranges from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$
- Graph B ranges from 0 to π
- Graph C ranges from 0 to 2π (correct)
- Graph D ranges from 0 to π

Therefore, $n(x) = 2\arccos(x)$ is graph C

B) For $m(x) = \arctan(\frac{x}{2}) + \pi$:

Range is $(\frac{\pi}{2}, \frac{3\pi}{2})$

C) For $f(x) = \frac{1}{2}\sin(x)$ on $[-\frac{\pi}{2}, \frac{\pi}{2}]$:

Domain of f^{-1} is $[-\frac{1}{2}, \frac{1}{2}]$

D) Range of f^{-1} is $[-\frac{\pi}{2}, \frac{\pi}{2}]$

(14) Solutions:

A) For $g(x) = 3\csc(x + 2) - 1$:

Let's track how each transformation affects the range:

1) The range of $\csc(x)$ is $(-\infty, -1] \cup [1, \infty)$

2) Horizontal shift: $\csc(x + 2)$ • Shifts the graph 2 units left • Range remains $(-\infty, -1] \cup [1, \infty)$

3) Vertical dilation: $3\csc(x + 2)$ • All output values are tripled • Range becomes $(-\infty, -3] \cup [3, \infty)$

4) Vertical shift: $3\csc(x + 2) - 1$ • Shifts the graph down 1 unit • Range becomes $(-\infty, -4] \cup [2, \infty)$

B) For $f(x) = \frac{1}{10}\csc(x) - 1$:

Domain is $\{x | x \neq \pi n, n \in \mathbb{Z}\}$

C) Vertical asymptotes of $\sec(x)$ on $[0, 2\pi]$:

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

D) Vertical asymptotes of $\cot(x)$ on $[0, 2\pi]$:

$x = 0, \pi, 2\pi$

E) Solutions for $\csc(3x) = 0$ on $[0, \pi]$:

For $\csc(3x) = 0$, $\frac{1}{\sin(3x)} = 0$, which never happens. A fraction can only be zero if the numerator is zero.

F) Solutions for $\sec(3x) = 2$ on $[0, \pi]$:

$$\begin{aligned} \sec(3x) &= 2 \\ \frac{1}{\cos(3x)} &= 2 \\ \cos(3x) &= \frac{1}{2} \end{aligned}$$

If we let $a = 3x$, then $\cos(a) = \frac{1}{2}$ occurs when:

$$\begin{aligned} a &= \pm\frac{\pi}{3} + 2\pi k \text{ where } k \text{ is an integer} \\ 3x &= \pm\frac{\pi}{3} + 2\pi k \\ x &= \pm\frac{\pi}{9} + \frac{2\pi k}{3} \end{aligned}$$

On $[0, \pi]$, this occurs when: $x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$