Quiz 5 Solutions

(1) Solutions:

A) In Triangle I, to find the value of α in radians:

In a triangle, the sum of angles is π radians.

$\alpha + \frac{\pi}{7} + \frac{\pi}{2} = \pi$

$\alpha = \pi - \frac{\pi}{7} - \frac{\pi}{2}$

$\alpha = \pi - \frac{\pi}{7} - \frac{\pi}{2} = \frac{7\pi - \pi - 7\pi/2}{7} = \frac{7\pi - \pi - 3.5\pi}{7} = \frac{2.5\pi}{7} = \frac{5\pi}{14}$

B) In Triangle II, to find the length of side y:

This is an isosceles right triangle with two angles of $\frac{\pi}{4}$ and one angle of $\frac{\pi}{2}$.

Using the Pythagorean theorem: $y^2 + y^2 = 18^2$

$2y^2 = 324$

$y^2 = 162$

$y = \sqrt{162} = 9\sqrt{2}$

C) In Triangle III, to find the length of side m:

Using the Pythagorean theorem in the right triangle:

$10^2 + m^2 = 15^2$

$100 + m^2 = 225$

$m^2 = 125$

$m = \sqrt{125} = 5\sqrt{5}$

(3) Solutions:

A) The y-coordinate of point Q:

In the right triangle shown in the figure, $\sin(\theta) = 0.7$ represents the ratio of the opposite side (the y-coordinate of point Q) to the hypotenuse (which is 5).

$\sin(\theta) = \frac{y}{5} = 0.7$

Therefore, $y = 5 \cdot 0.7 = 3.5$

B) $\sin\left(\frac{14\pi}{2}\right)$:

$\frac{14\pi}{2} = 7\pi$

$7\pi = 2\pi \cdot 3 + \pi$

Since sin has period 2π, $\sin(7\pi) = \sin(\pi) = 0$

C) $\sin\left(\frac{5\pi}{6}\right)$:

$\sin\left(\frac{5\pi}{6}\right) = \sin\left(\pi - \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

D) $\sin\left(\frac{4\pi}{3}\right)$:

$\sin\left(\frac{4\pi}{3}\right) = \sin\left(\pi + \frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

E) $\sin\left(-\frac{\pi}{3}\right)$:

$\sin\left(-\frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

(9) Solutions:

A) The equation using sine function:

From the graph, we can see:

- The amplitude is 1 (distance from midline to max/min)

- The midline is at $y = -1$

- The period is 2π - There is a phase shift of $\frac{\pi}{6}$ to the left

Therefore: $j(x) = \sin\left(x + \frac{\pi}{6}\right) - 1$

B) The equation using cosine function:

We can convert using the identity $\sin\left(x + \frac{\pi}{6}\right) = \cos\left(x + \frac{\pi}{6} - \frac{\pi}{2}\right) = \cos\left(x - \frac{\pi}{3}\right)$

Therefore: $j(x) = \cos\left(x - \frac{\pi}{3}\right) - 1$

C) The midline equation:

The midline is the horizontal line passing through the middle of the oscillation.

Midline equation: $y = -1$

D) The amplitude:

The amplitude is the distance from the midline to either the maximum or minimum value.

Amplitude $= 1$

(11) Solutions:

A) For the Ferris wheel problem:

Given information:

$$\text{Diameter} = 34 \text{ feet} \Rightarrow \text{radius} = 17 \text{ feet}$$
$$\text{Lowest point} = 12 \text{ feet off the ground}$$
$$\text{Period} = 50 \text{ seconds}$$
$$\text{Horizontal Dilation} = \frac{50}{2\pi} = \frac{25}{\pi}$$

The midline (average height) $= 12 + 17 = 29$ feet
The amplitude $= 17$ feet

Since Mr. Laird starts at the lowest point at $t = 0$, we can use a cosine function reflected over the midlin:

$$h(t) = -17\cos\left(\frac{\pi}{25}t\right) + 29$$

B) Times when Mr. Laird is at a height of 29 feet:

$$29 = 29 - 17\cos\left(\frac{\pi}{25}t\right)$$
$$0 = -17\cos\left(\frac{\pi}{25}t\right)$$
$$\cos\left(\frac{\pi}{25}t\right) = 0$$

This occurs when $\frac{\pi}{25}t = \frac{\pi}{2} + n\pi$ for integer n

$$t = \frac{25}{2} + 25n = 12.5 + 25n$$

The first two times are $t = 12.5$ seconds and $t = 37.5$ seconds.

C) Height at $t = 25$ seconds:

$$\begin{aligned} h(25) &= 29 - 17\cos\left(\frac{\pi}{25} \cdot 25\right) \\ &= 29 - 17\cos(\pi) \\ &= 29 - 17 \cdot (-1) \\ &= 29 + 17 \\ &= 46 \text{ feet} \end{aligned}$$

(12) Solutions:

A) The function equation using cosine:

Given information:

$$\begin{aligned} \text{Highest temperature} &= 85\text{F at } t = 6.7 \text{ months} \\ \text{Lowest temperature} &= 55\text{F at } t = 6.7 - 6 = 0.7 \text{ months} \end{aligned}$$

Amplitude $= \frac{85-55}{2} = 15$F
Midline (average temperature) $= \frac{85+55}{2} = 70$F
Period $= 12$ months
Horizontal Dilation $= \frac{12}{2\pi} = \frac{6}{\pi}$
Since the maximum occurs at $t = 6.7$, the phase shift is 6.7 months (from the standard position where maximum would occur at $t = 0$)

$$\begin{aligned} T(t) &= 70 + 15\cos\left(\frac{2\pi}{12}(t - 6.7)\right) \\ &= 70 + 15\cos\left(\frac{\pi}{6}(t - 6.7)\right) \end{aligned}$$

B) Temperature on the last day of October ($t = 9.7$):

$$\begin{aligned} T(9.7) &= 70 + 15\cos\left(\frac{\pi}{6}(9.7 - 6.7)\right) \\ &= 70 + 15\cos\left(\frac{\pi}{6} \cdot 3\right) \\ &= 70 + 15\cos\left(\frac{\pi}{2}\right) \\ &= 70 + 15 \cdot 0 \\ &= 70\text{F} \end{aligned}$$

C) Times when temperature is increasing at an increasing rate:

Our temperature is increasing from $t = 0.7$ to $t = 6.7$, between our minimum and maximum points.

For the first half of this interval, the temperature is increasing at an increasing rate. For the second half of this interval, the temperature is increasing at a decreasing rate.

This is true for all sinusoidal functions, and is true for our function.

Therefore, the temperature is increasing at an increasing rate between $t = 0.7$ and $t = 3.7$ months. Additionally, the temperature will be increasing at an increasing rate for all intervals of the form $[0.7 + 12n, 3.7 + 12n]$ for $n \in \mathbb{Z}$.