Quiz 4 Solutions

(6) Solutions:

A) $\cos(-\pi/12) = \cos(\pi/12) = \frac{\sqrt{2}+\sqrt{6}}{4}$

B) $\cos(\pi/12) = \frac{\sqrt{2}+\sqrt{6}}{4}$

C) $\sin(11\pi/12) = \sin(\pi - \pi/12) = \sin(\pi)\cos(-\pi/12) - \cos(\pi)\sin(-\pi/12)$
$= 0 \cdot \cos(\pi/12) - (-1) \cdot (-\sin(\pi/12))$
$= \sin(\pi/12) = \frac{\sqrt{6}-\sqrt{2}}{4}$

D) $\sin(x + \pi/4) = \sin(x)\cos(\pi/4) + \cos(x)\sin(\pi/4)$
$= \sin(x) \cdot \frac{\sqrt{2}}{2} + \cos(x) \cdot \frac{\sqrt{2}}{2}$
$= \frac{\sqrt{2}}{2}(\sin(x) + \cos(x))$

E) $\sin(x + \pi/2) = \sin(x)\cos(\pi/2) + \cos(x)\sin(\pi/2)$
$= \sin(x) \cdot 0 + \cos(x) \cdot 1$
$= \cos(x)$

F) $\cos(x - \pi/3) = \cos(x)\cos(-\pi/3) - \sin(x)\sin(-\pi/3)$
$= \cos(x) \cdot \frac{1}{2} - \sin(x) \cdot (-\frac{\sqrt{3}}{2})$
$= \frac{1}{2}\cos(x) + \frac{\sqrt{3}}{2}\sin(x)$

(7) Solutions:
Consider the graph of $f(x) = \tan(x - \pi/2)$.

A) The period of $f(x) = \tan(x - \pi/2)$ is π, which is the period of the tangent function.

B) The frequency of $f(x)$ is $\frac{1}{\pi}$.

C) For $\tan(x - \pi/2)$, the asymptotes occur when $x - \pi/2 = \pi/2 + n\pi$.
Solving: $x = \pi + n\pi$.
In $[0, 2\pi]$, the asymptotes are at $x = 0, x = \pi$ and $x = 2\pi$.

D) Zeros occur when $\tan(x - \pi/2) = 0$, which happens when $x - \pi/2 = n\pi$.
Solving: $x = \pi/2 + n\pi$.
In $[0, 2\pi]$, the zeros are at $x = \pi/2$ and $x = 3\pi/2$.

E) Zeros in $[8\pi, 10\pi]$ occur at $x = \pi/2 + n\pi$ where n is an integer.
These are $x = 8\pi + \pi/2 = 17\pi/2$ and $x = 9\pi + \pi/2 = 19\pi/2$.

For function $g(x)$ with period 2π:

F) $g(7\pi/3) = g(\pi/3) = 1$

G) $g(8\pi/3) = g(2\pi/3) = -1$

H) $g(-4\pi/3) = g(-4\pi/3 + 2\pi) = g(2\pi/3) = -1$

(8) Solutions:
For the sinusoidal function $m(x)$ shown in the graph:

A) The period is 4 units, as shown in the graph (one complete cycle, maximum to minimum to maximum).

B) The frequency is $\frac{1}{4} = 0.25$.

C) The amplitude is 2 units (the function oscillates from -1 to 3, which means the amplitude is $\frac{3-(-1)}{2} = 2$).

D) The midline equation is $y = 1$ (the average of the maximum value 3 and minimum value -1).

E) At point Q where $x = 1$, the function is decreasing.

F) Neither. At point Q, the rate of change is not changing because point Q is an inflection point.

(10) Solutions:

A) For $f(x) = a\sin(b(x-c)) + d$:
Given: $(2,2)$ is a minimum and $(4,4)$ is a maximum.
Since the amplitude is $\frac{4-2}{2} = 1$, we have $|a| = 1$.
The horizontal distance between the minimum and maximum is 2, so the period is 4.
Our horizontal dilation factor is $\frac{2}{\pi} = 2$, so $b = \frac{\pi}{2}$.

B) The transformation from $p(x) = \sin(x)$ to $q(x) = 5\sin(x)$ is a vertical dilation by a factor of 5.

C) The period remains unchanged at 2π.

D) The amplitude increases from 1 to 5.

E) The midline remains unchanged at $y = 0$.

F) The transformation from $j(x) = \sin(x)$ to $k(x) = \sin(x/4)$ is a horizontal dilation by a factor of 4.

G) The period increases from 2π to 8π.

H) The amplitude remains unchanged at 1.

I) The midline remains unchanged at $y = 0$.