Spring 2025 Laird Homework 4

Question 1

(A) Write a sinusoidal function with a period of 5, an amplitude of 3, a midline of y=10, that passes through (2,10).

Question 2

The number of minutes of daylight per day for a certain city can be modeled by the function D given by $D(t) = 160\cos\left(\frac{2\pi}{365}(t-172)\right) + 729$, where t is the day of the year for $1 \leq t \leq 365$.

(A) Describe whether the daylight minutes are increasing or decreasing on day 150. Are they increasing or decreasing at an increasing or decreasing rate?

(B) Which day of the year gets the most daylight?

Question 3

Consider the function $f(x) = \sin(x+c)$, where c is a constant.

(A) For which values of c, if any, is $f(x)$ an even function?

(B) For which values of c, if any, is $f(x)$ an odd function?

Question 4

The height of a point on a dolphin swimming in the ocean can be modeled by a sinusoidal function h. For time t in seconds, $h(t)$ represents the height relative to the water's surface: positive values indicate the dolphin is above the surface, and negative values indicate it is below.

The dolphin jumps to a maximum height of 2.5 feet above the surface, then dives to a minimum height of 2.5 feet below the surface before returning to the surface and repeating the cycle. It takes the dolphin 3 seconds to reach its maximum height from the surface.

(A) Find a sinusoidal function $h(t)$ that models the height of the point on the dolphin, assuming that the dolphin is at the surface of the water (h = 0) when $t = 0$. Write your function using sine.

Question 5

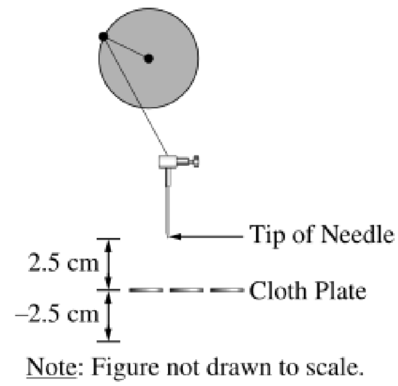
The Umpqua River flows into Half Moon Bay. According to local tide measurements, on New Year's Day, "high tide" occurred at approximately 3:15am when the water level reached 0.7 feet above sea level. On that same day, "low tide" occurred at approximately 9:30am when the water level reached -0.3 feet relative to sea level. Assume that midnight is represented by $t = 0$ hours and that one full tidal cycle takes 24 hours.

(A) Write an equation for the tidal height $h(t)$ in the form of a sinusoidal function.

(B) According to your model, at what two times on New Year's Day was the water level exactly at sea level (0 feet)?

Question 6

The vertical motion of the tip of a needle on a sewing machine is periodic with respect to time and can be modeled by a sinusoidal function. The figure shows the location of the tip of the needle in relation to a cloth plate. The tip of the needle moves straight up and down above and below the cloth plate.



The table below gives consecutive maximum and minimum displacements of the tip at two times.

Time t (seconds)	Displacement of Tip from Cloth Plate (centimeters)
0	2.5
$\frac{1}{20}$	-2.5

The function d models the displacement, in centimeters, of the tip of the needle from the cloth plate at time t, in seconds.

(A) Find a sinusoidal function $d(t)$ that models the displacement of the needle. Express your answer using the cosine function.

Question 7

The function $f(x)$ is defined by $f(x) = \cos(x)$.

Function $g(x)$ is a horizontal dilation of $f(x)$ by a factor of $\frac{1}{2}$.

Function $h(x)$ is a phase shift of $g(x)$ by $-\frac{\pi}{4}$ units.

(A) Find ALL local minima of $h(x)$.

Question 8

The number of hours of daylight in New York can be modeled by a sinusoidal function D. The maximum number of daylight hours is 14.5 and occurs on June 21. The minimum number of daylight hours is 9.5 and occurs on December 21.

Let $D(t)$ represent the number of hours of daylight in New York on day t, where $t = 0$ corresponds to January 1.

(A) Write an equation for $D(t)$ in the form $D(t) = a\sin(b(t - c)) + d$.

(B) According to your model, how many hours of daylight are there on April 1 (day 91)?

(C) According to your model, on what day of the year are there exactly 12 hours of daylight?

(D) During what dates is the number of daylight hours increasing at an increasing rate?

Question 9

The function $f(x) = 2\sin(4x)$ is defined on the interval $0 < x < 100$.

(A) How many complete periods does this function complete on the given interval? You may use a calculator to help you.