Spring 2025 Laird Homework 2 Solutions

Please note that a calculator is not required for any of the following questions, and is discouraged. If you are using a calculator, you are doing it wrong. Use your unit circle knowledge.

1. Find the exact value of $\sin(\frac{5\pi}{12})$. Do not give a decimal approximation!
2. Find the exact value of $\cos(\frac{7\pi}{12})$. Do not give a decimal approximation!
3. Find the exact value of $\cos(\frac{-7\pi}{12})$. Do not give a decimal approximation!
4. Let x and y be angles in standard position. If $\sin(x) = \frac{3}{5}$ and $\cos(y) = -\frac{1}{2}$ where x is in Quadrant I and y is in Quadrant II, find:
   (a) $\sin(x + y)$
   (b) $\cos(x - y)$
5. Find all values of x in the interval $[0, 2\pi]$ that satisfy both inequalities:
   (a) $\cos(x) < -\frac{\sqrt{2}}{2}$
   (b) $2\sin(x) < 1$
6. Find all values of x in the interval $[-\pi, \pi]$ that satisfy both inequalities:
   (a) $\sin(x) > \frac{\sqrt{3}}{2}$
   (b) $\cos(x) > -\frac{1}{2}$
7. Find all values of x in the interval $[10\pi, 14\pi]$ that satisfy $\sin(x) = \frac{1}{2}$
8. Find all values of x in the interval $[-7\pi, -3\pi]$ that satisfy $\cos(x) = -\frac{\sqrt{2}}{2}$
9. At what x-value(s) in $[0, 3\pi]$ does $\tan(x)$ have vertical asymptotes?
10. How many times does $y = \sin(x)$ intersect $y = \cos(x)$ in the interval $[0, 2\pi]$? (*Hint: Practice graphing sin and cos*)

Solutions

1. $\sin(\frac{5\pi}{12})$:
   (a) Break into sum: $\frac{5\pi}{12} = \frac{\pi}{4} + \frac{\pi}{6}$
   (b) Apply sum formula: $\sin(\frac{\pi}{4})\cos(\frac{\pi}{6}) + \cos(\frac{\pi}{4})\sin(\frac{\pi}{6})$
   (c) Substitute known values: $\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$
   (d) Simplify: $\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$
2. $\cos(\frac{7\pi}{12})$:
   (a) Break into difference: $\frac{7\pi}{12} = \frac{3\pi}{4} - \frac{\pi}{6}$
   (b) Apply difference formula: $\cos(\frac{3\pi}{4})\cos(\frac{\pi}{6}) + \sin(\frac{3\pi}{4})\sin(\frac{\pi}{6})$
   (c) Substitute known values: $-\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$
   (d) Simplify: $-\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{-\sqrt{6}+\sqrt{2}}{4}$
3. $\cos(\frac{-7\pi}{12})$:
   (a) $\cos(\alpha) = \cos(-\alpha)$
   (b) $\cos(\frac{-7\pi}{12}) = \cos(\frac{7\pi}{12})$
   (c) $\cos(\frac{7\pi}{12}) = \frac{-\sqrt{6}+\sqrt{2}}{4}$
4. Given $\sin(x) = \frac{3}{5}$ (Quadrant I) and $\cos(y) = -\frac{1}{2}$ (Quadrant II):
   (a) First find missing parts:
      - For x: $\cos(x) = \frac{4}{5}$ (using Pythagorean identity)
      - For y: $\sin(y) = \sqrt{1-\cos^2(y)} = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$
   (b) $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$
   $$= \frac{3}{5}(-\frac{1}{2}) + \frac{4}{5}(\frac{\sqrt{3}}{2}) = \frac{4\sqrt{3}-3}{10}$$
   (c) $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
   $$= \frac{4}{5}(-\frac{1}{2}) + \frac{3}{5}(\frac{\sqrt{3}}{2}) = \frac{3\sqrt{3}-4}{10}$$
5. For x in $[0, 2\pi]$:
   (a) $\cos(x) < -\frac{\sqrt{2}}{2}$ means x is in the second or third quadrant, specifically $\frac{3\pi}{4} < x < \frac{5\pi}{4}$
   (b) $2\sin(x) < 1$ means $\sin(x) < \frac{1}{2}$, so $x < \frac{\pi}{6}$ or $x > \frac{5\pi}{6}$
   (c) The intersection of these intervals in $[0, 2\pi]$ is $(\frac{5\pi}{6}, \frac{5\pi}{4})$
6. For x in $[-\pi, \pi]$:
   (a) $\sin(x) > \frac{\sqrt{3}}{2}$ means x is near the peak of sine curve: $\frac{\pi}{3} < x < \frac{2\pi}{3}$ (Quadrant I and II)
   (b) Considering only Quadrant I and II, $\cos(x) > -\frac{1}{2}$ means x is less than $\frac{2\pi}{3}$
   (c) The intersection is $(\frac{\pi}{3}, \frac{2\pi}{3})$
7. For $[10\pi, 14\pi]$, solving $\sin(x) = \frac{1}{2}$:
   (a) Reference angle is $\frac{\pi}{6}$

(b) In one period $[0, 2\pi]$, solutions are $\frac{\pi}{6}$ and $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$

(c) Add multiples of 2π until in range:

- From $\frac{\pi}{6}$: $10\pi + \frac{\pi}{6}$, $12\pi + \frac{\pi}{6}$
- From $\frac{5\pi}{6}$: $10\pi + \frac{5\pi}{6}$, $12\pi + \frac{5\pi}{6}$

8. For $[-7\pi, -3\pi]$, solving $\cos(x) = -\frac{\sqrt{2}}{2}$:

   (a) Reference angle is $\frac{3\pi}{4}$

   (b) In one period $[0, 2\pi]$, solutions are $\frac{3\pi}{4}$ and $\frac{5\pi}{4}$

   (c) Add multiples of -2π until in range:

   - From $\frac{3\pi}{4}$: $-6\pi + \frac{3\pi}{4}$, $-4\pi + \frac{3\pi}{4}$
   - From $\frac{5\pi}{4}$: $-8\pi + \frac{5\pi}{4}$, $-6\pi + \frac{5\pi}{4}$

9. Vertical asymptotes of $\tan(x)$ in $[0, 3\pi]$:

   (a) Occur when $\cos(x) = 0$

   (b) This happens at $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

10. The intersections of $y = \sin(x)$ and $y = \cos(x)$ in $[0, 2\pi]$:

   (a) Set equal: $\sin(x) = \cos(x)$

   (b) Rearrange: $\sin(x) - \cos(x) = 0$

   (c) Factor: $\sqrt{2}\sin(x - \frac{\pi}{4}) = 0$

   (d) Solve: $x - \frac{\pi}{4} = 0$ or $x - \frac{\pi}{4} = \pi$

   (e) Therefore $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$

   (f) Verify these are in $[0, 2\pi]$: Yes