Spring 2025 Laird Homework 1 Solutions

1. In a right triangle, if one leg is 15 units and the hypotenuse is 17 units, find:
   (a) The length of the other leg
   (b) The sine of the angle between the 15-unit leg and the hypotenuse
   (c) The sine of the angle between the unknown leg and the hypotenuse
2. An equilateral triangle has sides of length 8 units.
   (a) Find the height of the triangle
   (b) Find the area of the triangle
   (c) What is the measure of each angle in radians?
3. Consider a triangle where side $a = 7$ units, angle $A = \frac{5\pi}{6}$ radians, and angle $B = \frac{\pi}{12}$ radians.
   (a) Find side b and side c
4. In a right triangle, if the angle between the adjacent side and the hypotenuse is $\frac{\pi}{4}$ radians and the opposite side is 6 units:
   (a) Find the length of the adjacent side
   (b) Find the length of the hypotenuse
5. In the xy-plane an angle in standard position measures $\frac{\pi}{6}$ radians. A circle centered at the origin has a radius of 10 units. What is the y-coordinate of the point where the terminal ray of the angle intersects the circle?
6. In the xy-plane an angle in standard position measures $\frac{5\pi}{6}$ radians. A circle centered at the origin has a radius of 10 units. What is the y-coordinate of the point where the terminal ray of the angle intersects the circle?

Solutions

1. In the right triangle with leg 15 and hypotenuse 17:

   (a) For the unknown leg (call it x):
      - Using the Pythagorean theorem: $15^2 + x^2 = 17^2$
      - $225 + x^2 = 289$
      - $x^2 = 64$
      - $x = 8$ (since length is positive)

   (b) For sin of angle between 15-unit leg and hypotenuse:
      - $\sin = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{8}{17}$

   (c) For sin of angle between 8-unit leg and hypotenuse:
      - $\sin = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{15}{17}$

2. For the equilateral triangle with sides of 8 units:

   (a) Height (h):
      - Using 30-60-90 triangle formed by height
      - Half of base is 4 units
      - $h = 4\sqrt{3}$ units

   (b) Area:
      - $A = \frac{1}{2} \cdot \text{base} \cdot \text{height}$
      - $A = \frac{1}{2} \cdot 8 \cdot 4\sqrt{3}$
      - $A = 16\sqrt{3}$ square units

   (c) Each angle measures:
      - $\frac{\pi}{3}$ radians (since equilateral triangles have 60° angles)

3. For triangle with $a = 7$, $A = \frac{5\pi}{6}$, $B = \frac{\pi}{12}$:

   (a) Finding sides b and c:
      - First find angle C: $C = \pi - A - B = \pi - \frac{5\pi}{6} - \frac{\pi}{12} = \frac{\pi}{12}$
      - Using law of sines:
      $$\frac{b}{\sin(B)} = \frac{7}{\sin(A)}$$
      - $b = 7 \cdot \frac{\sin(\frac{\pi}{12})}{\sin(\frac{5\pi}{6})} = 7 \cdot \frac{\sin(\frac{\pi}{12})}{\frac{1}{2}} = 14\sin(\frac{\pi}{12})$
      - Similarly for c:
      $$\frac{c}{\sin(C)} = \frac{7}{\sin(A)}$$
      - $c = 7 \cdot \frac{\sin(\frac{\pi}{12})}{\sin(\frac{5\pi}{6})} = 14\sin(\frac{\pi}{12}) \approx 3.623$

4. For right triangle with angle $\frac{\pi}{4}$ and opposite side 6:

   (a) Adjacent side:
      - Using $\tan(\frac{\pi}{4}) = \frac{\text{opposite}}{\text{adjacent}} = 1$
      - Therefore adjacent = opposite = 6 units

   (b) Hypotenuse:
      - Using Pythagorean theorem: $6^2 + 6^2 = c^2$
      - $72 = c^2$
      - $c = 6\sqrt{2}$ units

5. For angle $\frac{\pi}{6}$ and radius 10:
   - y-coordinate $= r\sin(\frac{\pi}{6})$
   - $= 10 \cdot \frac{1}{2}$
   - $= 5$ units
6. For angle $\frac{5\pi}{6}$ and radius 10:
   - y-coordinate $= r\sin(\frac{5\pi}{6})$
   - $= 10 \cdot \frac{1}{2}$
   - $= 5$ units