Multiple-Choice Section
Course Framework Alignment and Rationales

Question 1

Skill	Learning Objective	Topic
1.A: Solve equations and inequalities represented analytically, with and without technology.	2.13.A: Solve exponential and logarithmic equations and inequalities.	2.13: Exponential and Logarithmic Equations and Inequalities

(A)	Correct. To find the x-coordinate of the point of intersection of the graphs of the two functions, solve the equation $4^x = 16^{x+2}$. Because $16^{x+2} = 4^{2(x+2)}$, the equation can be rewritten as $4^x = 4^{2(x+2)}$. Therefore, the exponents are equal and solving $x = 2(x+2)$ results in $x = 2x + 4$, which means $x = -4$.
(B)	Incorrect. This response is the result of rewriting 16^{x+2} as 4^{2x+2} and solving the equation $4^x = 4^{2x+2}$, which results in $x = -2$.
(C)	Incorrect. This response is the result of rewriting 16^{x+2} as 4^{2x}, ignoring the +2 in the original exponent, and solving the equation $4^x = 4^{2x}$, which results in $x = 0$.
(D)	Incorrect. This response is the result of thinking that the equation to solve is $4^{2x} = 16^{x+2}$ and concluding that $2x = x + 2$, which results in $x = 2$.

Question 2

Skill	Learning Objective	Topic
1.A: Solve equations and inequalities represented analytically, with and without technology.	2.9.A: Evaluate logarithmic expressions.	2.9: Logarithmic Expressions

(A)	Incorrect. This response may result from thinking that the input value of 16 needs to be multiplied by 2 because of the base 2 logarithmic function or from thinking that 2^4 is equal to 32.
(B)	Correct. The function f yields an output value of 4 when $\log_2 x = 4$. This is equivalent to $2^4 = x$, and the input value is $x = 16$.
(C)	Incorrect. This response is the result of using 4 as the input value such that $f(x) = \log_2 4$. This is equivalent to $2^y = 4$, which results in $y = 2$.
(D)	Incorrect. This response is the result of thinking that the square root of the output value 4 is 2, so the input value must be $\frac{1}{2}$.

Question 3

Skill	Learning Objective	Topic
1.A: Solve equations and inequalities represented analytically, with and without technology.	3.3.A: Determine coordinates of points on a circle centered at the origin.	3.3: Sine and Cosine Function Values

(A)	Incorrect. These are the values where $2\sin\theta = 1$ or $\sin\theta = \frac{1}{2}$ for $0 \leq \theta < 2\pi$.
(B)	Incorrect. These are the values where $2\cos\theta = 1$ or $\cos\theta = \frac{1}{2}$ for $0 \leq \theta < 2\pi$.
(C)	Incorrect. These are the values where $2\cos\theta = -1$ or $\cos\theta = -\frac{1}{2}$ for $0 \leq \theta < 2\pi$.
(D)	Correct. Solving $2\sin\theta = -1$ means finding the values for θ, for $0 \leq \theta < 2\pi$, where $\sin\theta = -\frac{1}{2}$. Sine is negative in Quadrants III and IV, and knowing that $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ can be used to determine that $\theta = \frac{7\pi}{6}$ and $\theta = \frac{11\pi}{6}$.

Question 4

Skill	Learning Objective	Topic
1.B: Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context.	2.12.A: Rewrite logarithmic expressions in equivalent forms.	2.12: Logarithmic Function Manipulation

(A)	Incorrect. This response is the result of thinking that the product property for logarithms can be applied to rewrite the expression as a sum.
(B)	Incorrect. This response is the result of thinking that the power property for logarithms involves taking the logarithm of the exponent.
(C)	Correct. Using the power property for logarithms, $\log_3(x^5)$ can be rewritten as $5\log_3 x$.
(D)	Incorrect. This response is the result of thinking that this is how the change of base property for logarithms is applied. Using the change of base property, $\log_3(x^5) = \frac{\log_a(x^5)}{\log_a 3}$, where $a > 0$ and $a \neq 1$.

Question 5

Skill	Learning Objective	Topic
3.A: Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools.	2.3.A: Identify key characteristics of exponential functions.	2.3: Exponential Functions

(A)	Incorrect. This describes the behavior of an exponential function $f(x) = ab^x$ where a is positive and $b > 1$.
(B)	Incorrect. This describes the behavior of an exponential function $f(x) = ab^x$ where a is negative and $0 < b < 1$.
(C)	Incorrect. This describes the behavior of an exponential function $f(x) = ab^x$ where a is positive and $0 < b < 1$.
(D)	Correct. The general form of an exponential function is $f(x) = ab^x$, where $a \neq 0$ and $b > 0$ with $b \neq 1$. Consider $y = 3 \cdot 4^x$. Because a is positive and $b > 1$, this exponential function is always increasing, and its graph is always concave up. Because of the negative sign with -3, h is a reflection of the graph of $y = 3 \cdot 4^x$ over the x-axis. Therefore, h is always decreasing, and its graph is always concave down.

Question 6

Skill	Learning Objective	Topic
1.C: Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.	1.10.A: Determine holes in graphs of rational functions.	1.10: Rational Functions and Holes

(A)	Incorrect. For this function, 2 is a zero of the polynomial in the numerator and a zero of the polynomial in the denominator. Both zeros have the same multiplicity. Therefore, there is a hole in the graph at $x = 2$. However, the graph of this function has a horizontal asymptote at $y = 6$. Input values of f sufficiently close to 2 do not correspond to output values arbitrarily close to 6.
(B)	Correct. The given information indicates that the hole in the graph of f is located at the point with coordinates $(2, 6)$. The corresponding mathematical notation is $\lim_{x \to 2} f(x) = 6$. Therefore, 2 must be a zero of the polynomial in the numerator and a zero of the polynomial in the denominator. To construct an expression for $f(x)$, the multiplicity of the zero 2 in the numerator must be greater than or equal to the multiplicity of the zero 2 in the denominator. For all values except $x = 2$, $\frac{(x-2)}{(x-2)} = 1$. Therefore, with $\frac{(x-2)}{(x-2)}$ as one factor, the other factor of f must have output values that are arbitrarily close to 6 for input values close to 2. When x is sufficiently close to 2, $\frac{(x+4)}{(x-1)}$ is arbitrarily close to 6.
(C)	Incorrect. The graph of this function has a hole at $x = 6$. For this function, input values of f sufficiently close to 6 correspond to output values arbitrarily close to 2. The corresponding mathematical notation is $\lim_{x \to 6} f(x) = 2$.
(D)	Incorrect. Evaluating this function for $x = 2$ results in the output value $\frac{3 \cdot 8}{1 \cdot 4} = \frac{24}{4} = 6$. However, the graph of this function does not have any holes.

Question 7

Skill	Learning Objective	Topic
2.A: Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.	1.1.B: Construct a graph representing two quantities that vary with respect to each other in a contextual scenario.	1.1: Change in Tandem

(A)	Incorrect. This container would correspond to the graph of a linear function with positive slope because the diameter is constant and the water is pouring into the container at a constant rate.
(B)	Correct. Because the diameter of the container decreases until the middle of the container, then increases, this container corresponds to the behavior shown in the graph. The graph is most steep in the middle portion of the graph with the relationship between the two quantities depth and time.
(C)	Incorrect. Because the diameter of the container increases until the middle of the container, this container will take longer to fill with water. The graph that corresponds to this container will be less steep in the middle portion of the graph with the relationship between the two quantities depth and time.
(D)	Incorrect. Because the diameter of the container is changing between decreasing and increasing multiple times, the graph that corresponds to this container will be more steep and less steep in multiple places. The relationship between the two quantities depth and time will result in a graph where there is more change between the two quantities than what the given graph demonstrates.

Question 8

Skill	Learning Objective	Topic
1.C: Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.	2.5.B: Apply exponential models to answer questions about a data set or contextual scenario.	2.5: Exponential Function Context and Data Modeling

(A)	Incorrect. This is a linear model that is not appropriate for the context of population decreasing by the same percent per year.
(B)	Incorrect. This is a linear model that is not appropriate for the context of population decreasing by the same percent per year.
(C)	Correct. Based on the context of population decreasing by the same percent per year, this is an exponential decay function model of the form ab^x. The initial value, a, is 23,144 at time $t = 0$ years. The base of the exponent, b, can be understood as a growth factor of 96% per year, which corresponds to a decrease of 4% per year. Therefore, an appropriate model for the population as a function of time t, $0 \le t \le 10$, is given by $23{,}144(0.96)^t$.
(D)	Incorrect. This is an exponential growth function model with a growth factor of 104% per year. This would be an appropriate model if the population were increasing at rate of 4% per year during each of the next 10 years.

Question 9

Skill	Learning Objective	Topic
1.B: Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context.	3.12.B: Rewrite trigonometric expressions in equivalent forms with sine and cosine sum identities.	3.12: Equivalent Representations of Trigonometric Functions

(A)	Incorrect. This response is the result of thinking that the numerator and denominator of $\frac{\sec^2 x - 1}{\sec^2 x}$ can be rewritten in an equivalent form as $\frac{\tan^2 x}{\sin^2 x}$.
(B)	Incorrect. This response is the result of thinking that the numerator and denominator of $\frac{\sec^2 x - 1}{\sec^2 x}$ can be rewritten in an equivalent form as $\frac{\tan^2 x}{\cos^2 x}$.
(C)	Correct. An equivalent form of $\frac{\sec^2 x - 1}{\sec^2 x}$ is found using the Pythagorean identity and the definitions of the trigonometric functions. $\frac{\sec^2 x - 1}{\sec^2 x} = \frac{\tan^2 x}{\frac{1}{\cos^2 x}} = \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}}$, which is equal to $\frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{1} = \sin^2 x$.
(D)	Incorrect. This response is the result of rewriting $\frac{\sec^2 x - 1}{\sec^2 x}$ as $\frac{\sec^2 x}{\sec^2 x} - \frac{1}{\sec^2 x}$, then thinking this is equivalent to $1 - \sin^2 x = \cos^2 x$.

Question 10

Skill	Learning Objective	Topic
1.B: Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context.	3.13.A: Determine the location of a point in the plane using both rectangular and polar coordinates.	3.13: Trigonometry and Polar Coordinates

(A)	Correct. The coordinates of a point in the polar coordinate system, (r, θ), can be converted to coordinates in the rectangular coordinate system, (x, y), using $x = r\cos\theta$ and $y = r\sin\theta$. Therefore, $x = 1\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$ and $y = 1\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$.
(B)	Incorrect. This response has an incorrect sign for the x-coordinate that results from thinking that $x = 1\cos\left(\frac{5\pi}{6}\right)$ is equal to $\frac{\sqrt{3}}{2}$.
(C)	Incorrect. This response is the result of interchanging the values of the x- and y-coordinates.
(D)	Incorrect. This response is the result of interchanging the values of the x- and y-coordinates and having an incorrect sign for $1\cos\left(\frac{5\pi}{6}\right)$.

Question 11

Skill	Learning Objective	Topic
3.B: Apply numerical results in a given mathematical or applied context.	2.5.A: Construct a model for situations involving proportional output values over equal-length input-value intervals.	2.5: Exponential Function Context and Data Modeling

(A)	Correct. Because this is an exponential decay function, the output values are proportional over equal-length input-value intervals. Using the labeled points $(2, 10)$ and $(3, 5)$ and the proportionality of the output values, it can be concluded that other points on the graph include $(1, 20)$ and $(0, 40)$.
(B)	Incorrect. This may result from mixing the behavior of a linear function and the behavior of an exponential decay function when determining the y-coordinate where $x = 0$.
(C)	Incorrect. This is the result of treating the graph as a linear function that passes through points $(2, 10)$ and $(3, 5)$.
(D)	Incorrect. This is the result of treating the graph as a linear function that passes through points $(2, 10)$ and $(3, 5)$ and finding the y-coordinate of the point where $x = 1$.

Question 12

Skill	Learning Objective	Topic
3.C: Support conclusions or choices with a logical rationale or appropriate data.	1.5.A: Identify key characteristics of a polynomial function related to its zeros when suitable factorizations are available or with technology.	1.5: Polynomial Functions and Complex Zeros

(A)	Incorrect. If the average rates of change of g over any length input-value interval were constant, the function would be linear. The average rates of change of g over the consecutive intervals $[-1, 0]$, $[0, 1]$, $[1, 2]$, and $[2, 3]$ are 1, 3, 5, and 7, respectively, which are not constant.
(B)	Incorrect. The function model is correct. The reasoning is not correct. The average rates of change of g over the consecutive intervals $[-1, 0]$, $[0, 1]$, $[1, 2]$, and $[2, 3]$ are 1, 3, 5, and 7, respectively, which are not constant.
(C)	Incorrect. The reasoning is correct. As a result, g is best modeled by a quadratic function. The successive 1st differences are not constant; therefore, the function model is not linear.
(D)	Correct. The degree of the polynomial function is equal to the least value n for which the successive nth differences are constant. In this case, the successive 1st differences are 1, 3, 5, and 7, and the successive 2nd differences are 2, 2, and 2. Therefore, g is best modeled by a quadratic function.

Question 13

Skill	Learning Objective	Topic
1.C: Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.	2.13.A: Solve exponential and logarithmic equations and inequalities.	2.13: Exponential and Logarithmic Equations and Inequalities

(A)	Incorrect. This response is the result of reversing the composition and finding $(g \circ f)(t) = g(f(t)) = 7\ln(e^t) = 7t$ or thinking that $h(t) = (f \circ g)(t) = f(g(t)) = e^{7\ln t}$ is equivalent to $7t$.
(B)	Correct. Using function composition, properties of logarithms, and the inverse relationship between the exponential function with the natural base e and the natural logarithm function, $h(t) = (f \circ g)(t) = f(g(t)) = e^{7\ln t} = e^{\ln(t^7)} = t^7$.
(C)	Incorrect. This response is the result of thinking that $h(t) = (f \circ g)(t) = f(g(t)) = e^{7\ln t}$ is equivalent to $e^7 \cdot e^{\ln t} = e^7 \cdot t = te^7$.
(D)	Incorrect. This response is the result of reversing the composition and finding $(g \circ f)(t) = g(f(t)) = 7\ln(e^t)$, then thinking this is equivalent to 7^t.

Question 14

Skill	Learning Objective	Topic
1.B: Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context.	1.11.C: Rewrite the repeated product of binomials using the binomial theorem.	1.11: Equivalent Representations of Polynomial and Rational Expressions

(A)	Incorrect. This is the coefficient of the x^2 term in the expanded polynomial. In expanded form, the sum of the exponents on the powers of -3 and x for each term of the polynomial is 5.
(B)	Correct. The binomial theorem utilizes the entries in a single row of Pascal's Triangle to expand $(x-3)^5$. The applicable row contains the entries 1, 5, 10, 10, 5, 1. In expanded form, $(x-3)^5 = 1\left((-3)^0\right)x^5 + 5\left((-3)^1\right)x^4 + 10\left((-3)^2\right)x^3 + 10\left((-3)^3\right)x^2 + 5\left((-3)^4\right)x^1 + 1\left((-3)^5\right)x^0$. Note that the sum of the exponents on the powers of -3 and x for each term of the polynomial is 5.
(C)	Incorrect. The coefficient of the x^3 term in the expanded polynomial is multiplied by the third value in the row of Pascal's Triangle containing the entries 1, 5, 10, 10, 5, 1. In expanded form, the sum of the exponents on the powers of -3 and x for each term of the polynomial is 5.
(D)	Incorrect. The coefficient of the x^3 term in the expanded polynomial is multiplied by the third value in the row of Pascal's Triangle containing the entries 1, 5, 10, 10, 5, 1.

Question 15

Skill	Learning Objective	Topic
2.B: Construct equivalent graphical, numerical, analytical, and verbal representations of functions that are useful in a given mathematical or applied context, with and without technology.	3.6.A: Identify the amplitude, vertical shift, period, and phase shift of a sinusoidal function.	3.6: Sinusoidal Function Transformations

(A)	Correct. The graph of $y = g(x) = 2\cos(\pi x) + 1$ has an amplitude of 2 units, a period of $\frac{1}{\pi} \cdot 2\pi = 2$ units, and a midline vertical shift of 1 unit up from $y = 0$.
(B)	Incorrect. This is the graph of $y = g(x) = \cos(\pi x) + 2$.
(C)	Incorrect. This is the graph of $y = g(x) = 2\cos(0.5\pi x) + 1$.
(D)	Incorrect. This is the graph of $y = g(x) = 2\sin(\pi x) + 1$.

Question 16

Skill	Learning Objective	Topic
2.A: Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.	1.2.A: Compare the rates of change at two points using average rates of change near the points.	1.2: Rates of Change

(A)	Incorrect. The rate of change of g at the relative maximum point A is 0.
(B)	Incorrect. The rate of change of g is negative at point B. However, the rate of change is not the least at this point for the four points identified.
(C)	Correct. The rate of change of g is least at the point of inflection C. The rate of change is negative at this point, and the graph is steeper at point C than it is at point B. The rate of change of g is changing from decreasing to increasing at point C.
(D)	Incorrect. Point D is to the right of a relative minimum of g. The rate of change of g is positive at point D.

Question 17

Skill	Learning Objective	Topic
3.C: Support conclusions or choices with a logical rationale or appropriate data.	3.13.A: Determine the location of a point in the plane using both rectangular and polar coordinates.	3.13: Trigonometry and Polar Coordinates

(A)	Incorrect. Point A lies in Quadrant IV.
(B)	Incorrect. Point B lies in Quadrant III.
(C)	Incorrect. Point C lies in Quadrant I.
(D)	Correct. Point D lies in Quadrant II. $\theta = \frac{7\pi}{4}$ is located in Quadrant IV. Because $r < 0$, moving along a line segment 2 units in the negative direction from the origin has the point plotted in Quadrant II.

Question 18

Skill	Learning Objective	Topic
1.C: Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.	3.6.A: Identify the amplitude, vertical shift, period, and phase shift of a sinusoidal function.	3.6: Sinusoidal Function Transformations

(A)	Incorrect. This response is the result of thinking that the daily increase of 800 gallons is accounted for by a horizontal translation or phase shift of the graph of g by -8 units.
(B)	Correct. $g(t) + 8$ is a better model, where the 8 represents a daily increase of 800 gallons. The new function is a vertical translation of the graph of g by $+8$ units.
(C)	Incorrect. This response is the result of thinking that the daily increase of 800 gallons is accounted for by a horizontal dilation of the graph of g and differs in period by a factor of $\frac{1}{8}$.
(D)	Incorrect. This response is the result of thinking that the daily increase of 800 gallons is accounted for by a vertical dilation of the graph of g and differs in amplitude by a factor of 8.

Question 19

Skill	Learning Objective	Topic
1.A: Solve equations and inequalities represented analytically, with and without technology.	3.2.A: Determine the sine, cosine, and tangent of an angle using the unit circle.	3.2: Sine, Cosine, and Tangent

(A)	Incorrect. The difference between the two angle measures is $\frac{3\pi}{2}$, which is three-fourths of a revolution of a circle centered at the origin. Therefore, the two angles do not share a terminal ray.
(B)	Incorrect. This response is the result of thinking that two angle measures with the same absolute value must share a terminal ray.
(C)	Correct. Both angle measures are positive, which indicates a rotation from the positive x-axis in the counterclockwise direction. The two angles share a terminal ray because the difference between the angle measures is 2π, one full revolution of a circle centered at the origin.
(D)	Incorrect. The difference between the two angle measures is π, which is one-half of a revolution of a circle centered at the origin. Therefore, the two angles do not share a terminal ray.

Question 20

Skill	Learning Objective	Topic
1.B: Express functions, equations, or expressions in analytically equivalent forms that are useful in a given mathematical or applied context.	2.4.A: Rewrite exponential expressions in equivalent forms.	2.4: Exponential Function Manipulation

(A)	Incorrect. This is not an equivalent form for $h(x)$. This expresses h as a horizontal dilation of f.
(B)	Correct. Graphically, every horizontal translation of an exponential function, $g(x) = b^{(x+k)}$, is equivalent to a vertical dilation, $g(x) = b^x \cdot b^k$. In this case, an equivalent form for $h(x)$ is $h(x) = 3^{(x+2)} = 3^x \cdot 3^2 = 3^x \cdot 9$, which expresses h as a vertical dilation of f.
(C)	Incorrect. This is not an equivalent form for $h(x)$. This expresses h as a vertical dilation and reflection of f.
(D)	Incorrect. This is not an equivalent form for $h(x)$. This expresses h as a vertical translation of f.

Question 21

Skill	Learning Objective	Topic
3.C: Support conclusions or choices with a logical rationale or appropriate data.	1.3.B: Determine the change in the average rates of change for linear, quadratic, and other function types.	1.3: Rates of Change in Linear and Quadratic Functions

(A)	Incorrect. From the given information, g is decreasing on $3 < x < 7$. Because the average rate of change over equal-length input-value intervals is decreasing for all small-length intervals, the graph of the function is concave down on $3 < x < 7$.
(B)	Incorrect. From the given information, g is decreasing on $3 < x < 7$. Because the average rate of change over equal-length input-value intervals is decreasing for all small-length intervals, the graph of the function is concave down on $3 < x < 7$.
(C)	Correct. From the given information, g is decreasing on $3 < x < 7$. Because the average rate of change over equal-length input-value intervals is decreasing for all small-length intervals, the graph of the function is concave down on $3 < x < 7$.
(D)	Incorrect. From the given information, g is decreasing on $3 < x < 7$. Because the average rate of change over equal-length input-value intervals is decreasing for all small-length intervals, the graph of the function is concave down on $3 < x < 7$.

Question 22

Skill	Learning Objective	Topic
1.C: Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.	3.6.A: Identify the amplitude, vertical shift, period, and phase shift of a sinusoidal function.	3.6: Sinusoidal Function Transformations

(A)	Incorrect. This response is a horizontal translation or phase shift of the graph of f by $+\frac{1}{2}$ unit or $\frac{1}{2}$ unit right. This is an additive transformation $f\left(x-\frac{1}{2}\right)$.
(B)	Correct. A horizontal translation or phase shift of the graph of f by $+\frac{\pi}{2}$ units or $\frac{\pi}{2}$ units right is an additive transformation $f\left(x-\frac{\pi}{2}\right)$.
(C)	Incorrect. This response is a vertical translation of the graph of f by $-\frac{\pi}{2}$ units or $\frac{\pi}{2}$ units down. This is an additive transformation $f(x)-\frac{\pi}{2}$.
(D)	Incorrect. This response is a vertical translation of the graph of f by $\frac{\pi}{2}$ units or $\frac{\pi}{2}$ units up. This is an additive transformation $f(x)+\frac{\pi}{2}$.

Question 23

Skill	Learning Objective	Topic
3.A: Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools.	2.7.B: Construct a representation of the composition of two or more functions.	2.7: Composition of Functions

(A)	Correct. The input for C is time, in hours, and the output is temperature, in degrees Celsius. The input for P is temperature, in degrees Celsius, and the output is electricity usage, in kilowatts. Therefore, the input for composition function K is time, in hours, and the output is electricity usage, in kilowatts.
(B)	Incorrect. Both the input and output units of K are incorrect. The input is time, in hours, and the output is electricity usage, in kilowatts.
(C)	Incorrect. The input and output units of K are reversed. The input is time, in hours, and the output is electricity usage, in kilowatts.
(D)	Incorrect. The input units of K are incorrect, and the output units of K are correct. The input is time, in hours, and the output is electricity usage, in kilowatts.

Question 24

Skill	Learning Objective	Topic
2.A: Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.	3.2.A: Determine the sine, cosine, and tangent of an angle using the unit circle.	3.2: Sine, Cosine, and Tangent

(A)	Incorrect. This response is a result of not including the radius of the circle or treating the circle as the unit circle with radius 1.
(B)	Incorrect. This response is a result of using sine rather than cosine. The sine of θ is the ratio of the vertical displacement of Q from the x-axis to the distance between the origin and Q. This also does not include the radius of the circle or treats the circle as the unit circle with radius 1.
(C)	Correct. The cosine of θ is the ratio of the horizontal displacement of Q from the y-axis to the distance between the origin and Q. The distance between the origin and Q is 5, the radius. The value of θ, in radians, can be found by the ratio of the length of the arc $\overset{\frown}{PQ}$ to the radius of the circle. Therefore, $\theta = \frac{6}{5}$ and the horizontal displacement of Q from the y-axis is $5 \cdot \cos\theta = 5\cos\left(\frac{6}{5}\right)$.
(D)	Incorrect. This response is a result of using sine rather than cosine. The sine of θ is the ratio of the vertical displacement of Q from the x-axis to the distance between the origin and Q.

Question 25

Skill	Learning Objective	Topic
3.A: Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools.	3.11.A: Identify key characteristics of functions that involve quotients of the sine and cosine functions.	3.11: The Secant, Cosecant, and Cotangent Functions

(A)	Incorrect. The graph of the cotangent function has vertical asymptotes for domain values where $\tan x = 0$. These occur at $x = 0 \pm k\pi$, where k is an integer.
(B)	Incorrect. This is a horizontal translation, or phase shift, of the graph of $f(x) = \cot x$ by $+\frac{\pi}{2}$ units. The graph has vertical asymptotes at $x = \frac{\pi}{2} \pm k\pi$, where k is an integer.
(C)	Incorrect. This is a horizontal translation, or phase shift, of the graph of $f(x) = \cot x$ by $+\frac{\pi}{4}$ units. The graph has vertical asymptotes at $x = \frac{\pi}{4} \pm k\pi$, where k is an integer.
(D)	Correct. This is a horizontal translation, or phase shift, of the graph of $f(x) = \cot x$ by $-\frac{\pi}{4}$ units. The graph has vertical asymptotes at $x = -\frac{\pi}{4} \pm k\pi$, where k is an integer. Therefore, the graph of this function has a vertical asymptote at $x = -\frac{\pi}{4} + \pi = \frac{3\pi}{4}$.

Question 26

Skill	Learning Objective	Topic
3.C: Support conclusions or choices with a logical rationale or appropriate data.	1.10.A: Determine holes in graphs of rational functions.	1.10: Rational Functions and Holes

(A)	Incorrect. The numerator of function r can be factored so $r(x) = \frac{(x-2)(x+1)}{(x+1)^2(x-2)}$. The zeros of the numerator, -1 and 2, each have a multiplicity of 1. The zeros of the denominator, -1 and 2, have multiplicity of 2 and 1, respectively. Therefore, there is a hole at $x = 2$ only, and the graph of r has a vertical asymptote at $x = -1$.
(B)	Incorrect. The numerator of function r can be factored so $r(x) = \frac{(x-2)(x+1)}{(x+1)^2(x-2)}$. The zeros of the numerator, -1 and 2, each have a multiplicity of 1. The zeros of the denominator, -1 and 2, have multiplicity of 2 and 1, respectively. Therefore, there is a hole at $x = 2$ only, and the graph of r has a vertical asymptote at $x = -1$.
(C)	Correct. The numerator of function r can be factored so $r(x) = \frac{(x-2)(x+1)}{(x+1)^2(x-2)}$. The zeros of the numerator, -1 and 2, each have a multiplicity of 1. The zeros of the denominator, -1 and 2, have a multiplicity of 2 and 1, respectively. Therefore, there is a hole at $x = 2$ only, and the graph of r has a vertical asymptote at $x = -1$.
(D)	Incorrect. The numerator of function r can be factored so $r(x) = \frac{(x-2)(x+1)}{(x+1)^2(x-2)}$. The zeros of the numerator, -1 and 2, each have a multiplicity of 1. The zeros of the denominator, -1 and 2, have a multiplicity of 2 and 1, respectively. Therefore, there is a hole at $x = 2$ only, and the graph of r has a vertical asymptote at $x = -1$.

Question 27

Skill	Learning Objective	Topic
3.A: Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools.	2.6.B: Validate a model constructed from a data set.	2.6: Competing Function Model Validation

(A)	Incorrect. The residual is the difference between the actual, or observed, value and the predicted value from the model. The residual is positive when the predicted value is below the actual value and is too low. This indicates an underestimate at *A*. The residual is negative when the predicted value is above the actual value and is too high. This indicates an overestimate at *B*. Based on the absolute values of the residuals, the difference between the predicted value and the actual value is greater for *B* than with *A*. Therefore, there is greater error in the model with *B*.
(B)	Correct. The residual is the difference between the actual, or observed, value and the predicted value from the model. The residual is positive when the predicted value is below the actual value and is too low. This indicates an underestimate at *A*. The residual is negative when the predicted value is above the actual value and is too high. This indicates an overestimate at *B*. Based on the absolute values of the residuals, the difference between the predicted value and the actual value is greater for *B* than with *A*. Therefore, there is greater error in the model with *B*.
(C)	Incorrect. The residual is the difference between the actual, or observed, value and the predicted value from the model. The residual is positive when the predicted value is below the actual value and is too low. This indicates an underestimate at *A*. The residual is negative when the predicted value is above the actual value and is too high. This indicates an overestimate at *B*. Based on the absolute values of the residuals, the difference between the predicted value and the actual value is greater for *B* than with *A*. Therefore, there is greater error in the model with *B*.
(D)	Incorrect. The residual is the difference between the actual, or observed, value and the predicted value from the model. The residual is positive when the predicted value is below the actual value and is too low. This indicates an underestimate at *A*. The residual is negative when the predicted value is above the actual value and is too high. This indicates an overestimate at *B*. Based on the absolute values of the residuals, the difference between the predicted value and the actual value is greater for *B* than with *A*. Therefore, there is greater error in the model with *B*.

Question 28

Skill	Learning Objective	Topic
2.A: Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.	3.14.A: Construct graphs of polar functions.	3.14: Polar Function Graphs

(A)	Correct. For $\theta = 0$, the graph begins on the polar axis for $r = -2$. The point with polar coordinates $(-2, 0)$ is on the graph. For $\theta = \frac{\pi}{3}$, the point with polar coordinates $\left(2 - 4 \cdot \frac{1}{2}, \frac{\pi}{3}\right) = \left(0, \frac{\pi}{3}\right)$ is on the graph. For $\theta = \frac{\pi}{6}$, the point with polar coordinates $\left(2 - 4 \cdot \frac{\sqrt{3}}{2}, \frac{\pi}{6}\right) = \left(2 - 2\sqrt{3}, \frac{\pi}{6}\right)$ is on the graph.
(B)	Incorrect. These values produce only part of the graph.
(C)	Incorrect. This response is the result of thinking that the portion of the graph shown is Quadrant III and therefore must begin at $\theta = \pi$ and end at some value of θ that is greater than π.
(D)	Incorrect. This response is the result of thinking that the portion of the graph shown is Quadrant III and therefore must begin at $\theta = \pi$ and end at some value of θ that is greater than π.

Question 76

Skill	Learning Objective	Topic
2.A: Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.	1.3.A: Determine the average rates of change for sequences and functions, including linear, quadratic, and other function types.	1.3: Rates of Change in Linear and Quadratic Functions

(A)	Correct. The average rate of change over the closed interval $[1, 4]$ is the slope of the secant line from the point $(1, 16)$ to $(4, 10)$. The average rate of change is $\frac{f(4) - f(1)}{4 - 1} = \frac{10 - 16}{4 - 1} = \frac{-6}{3} = -2$.
(B)	Incorrect. This is the reciprocal of the average rate of change.
(C)	Incorrect. This response is the result of thinking that the average rate of change must be positive or using $f(1) - f(4)$ as the numerator.
(D)	Incorrect. This is the arithmetic mean of $f(1)$ and $f(4)$.

Question 77

Skill	Learning Objective	Topic
3.B: Apply numerical results in a given mathematical or applied context.	1.14.A: Construct a linear, quadratic, cubic, quartic, polynomial of degree n, or related piecewise-defined function model.	1.14: Function Model Construction and Application

(A)	Incorrect. This response is a result of calculating a linear regression to model the data and evaluating the model for $t = 18$.
(B)	Correct. Using a graphing calculator, a quartic regression of the data is calculated as approximately $y = 0.0036t^4 - 0.0833t^3 + 0.6898t^2 + 0.5833t + 1$, where t is the number of years since 2010. Store the regression model in the calculator without rounding the values of the coefficients, and use the stored model to evaluate for $t = 18$. The model predicts a length of 127.0 in thousands of kilometers for 2028.
(C)	Incorrect. This response is a result of calculating an exponential regression $y = ab^x$ to model the data and evaluating the model for $t = 18$. In Desmos, use log mode with the exponential regression to produce the same result as a handheld graphing calculator.
(D)	Incorrect. This response is a result of calculating the quartic regression to model the data and evaluating the model for $t = 28$, thinking that will produce the length in thousands of kilometers for 2028.

Question 78

Skill	Learning Objective	Topic
2.A: Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.	1.1.B: Construct a graph representing two quantities that vary with respect to each other in a contextual scenario.	1.1: Change in Tandem

(A)	Incorrect. This is an interval on which the graph of g is concave up and the rate of change of g is increasing.
(B)	Incorrect. This is an interval on which the graph of g is concave down and g is decreasing. The rate of change of g is also decreasing on this interval; however, there is another interval on which the rate of change of g is decreasing.
(C)	Incorrect. These are the intervals on which g is decreasing. The graph of g is concave up on $\left[t_A, t_B\right]$ because the rate of change of g is increasing on that interval.
(D)	Correct. These are the intervals on which the graph of g is concave down because the rate of change of g is decreasing.

Question 79

Skill	Learning Objective	Topic
2.A: Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.	3.8.A: Construct representations of the tangent function using the unit circle.	3.8: The Tangent Function

(A)	Incorrect. This response is the reciprocal of the slope of the line, which is equal to tan θ.
(B)	Incorrect. This response is the result of thinking that the slope of the line is equal to cos θ rather than tan θ.
(C)	Correct. Given the angle of measure θ in standard position and a unit circle centered at the origin, point P is where the terminal ray intersects the circle. Therefore, the tangent function gives the slope of the terminal ray, which is on the line that passes through the origin and P. The slope is $\tan\theta = -\frac{3}{4}$.
(D)	Incorrect. This response is the result of thinking that the slope of the line is equal to sin θ rather than tan θ.

Question 80

Skill	Learning Objective	Topic
1.C: Construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.	2.15.A: Determine if an exponential model is appropriate by examining a semi-log plot of a data set.	2.15: Semi-log Plots

(A)	Incorrect. If the semi-log plot appears to follow a linear pattern, the data are best modeled by an exponential function.
(B)	Incorrect. If the data are best modeled by an exponential growth function, the semi-log plot would appear to follow an increasing linear pattern.
(C)	Correct. If the semi-log plot appears to follow a decreasing linear pattern, the data are best modeled by an exponential decay function.
(D)	Incorrect. If the semi-log plot appears to follow a linear pattern, the data are best modeled by an exponential function.

Question 81

Skill	Learning Objective	Topic
3.B: Apply numerical results in a given mathematical or applied context.	1.14.C: Apply a function model to answer questions about a data set or contextual scenario.	1.14: Function Model Construction and Application

(A)	Correct. In function C, the value of b can be found by solving either $115 = \frac{1000 + b(10)}{10}$ or $65 = \frac{1000 + b(20)}{20}$. This results in $b = 15$. Using $C(x) = \frac{1000 + 15x}{x}$, the average rate of change of C on $[30, 40]$ is $\frac{C(40) - C(30)}{40 - 30} = -\$0.83$ per item. The average rate of change can also be found without solving for b. If $C(x) = \frac{1000 + bx}{x}$, then $\frac{C(40) - C(30)}{40 - 30} = \frac{\left(\frac{1000 + 40b}{40} - \left(\frac{1000 + 30b}{30}\right)\right)}{10}$, which simplifies to $\frac{3000 + 120b - 4000 - 120b}{120} \cdot \frac{1}{10} = \frac{-1000}{1200} = -\$0.83$ per item.
(B)	Incorrect. This response is the result of thinking that the average rate of change of C is the same for any interval of length 10. Using the given information with $x = 10$ and $x = 20$ results in calculating $\frac{65 - 115}{20 - 10} = -\$5$ per item.
(C)	Incorrect. This response is the result of finding the value of $C(40) - C(30)$ without dividing that difference by 10, the length of the interval, to find the average rate of change.
(D)	Incorrect. This response is the result of thinking that the average rate of change of C is the same for any interval of length 10. Using the given information with $x = 10$ and $x = 20$ results in calculating $65 - 115 = -\$50$ per item, without dividing that by 10, the length of the interval.

Question 82

Skill	Learning Objective	Topic
3.C: Support conclusions or choices with a logical rationale or appropriate data.	1.14.C: Apply a function model to answer questions about a data set or contextual scenario.	1.14: Function Model Construction and Application

(A)	Incorrect. Both the time t and the reasoning are incorrect. By examining the graph of P with the graphing calculator, the rate of change in the number of people inside the shopping center is a relative maximum at $t = 0.451$. This does not provide information about when the number of people inside the shopping center is at its maximum.
(B)	Incorrect. The time t is incorrect, and the reasoning is correct. By examining the graph of P with the graphing calculator, the rate of change in the number of people inside the shopping center is a relative maximum at $t = 0.451$. The rate of change in the number of people inside the shopping center is positive on an interval containing $t = 0.451$. This does not provide information about when the number of people inside the shopping center is at its maximum.
(C)	Incorrect. The time t is correct, and the reasoning is incorrect. By examining the graph of P with the graphing calculator, the rate of change in the number of people inside the shopping center is decreasing on an interval containing $t = 1.445$.
(D)	Correct. Both the time t and the reasoning are correct. By examining the graph of P with the graphing calculator, the number of people inside the shopping center will be at a maximum when the rate of change changes from positive to negative.

Question 83

Skill	Learning Objective	Topic
2.A: Identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology.	1.9.A: Determine vertical asymptotes of graphs of rational functions.	1.9: Rational Functions and Vertical Asymptotes

(A)	Incorrect. Based on the table, the rational function f is undefined at $x = -5$. Therefore, the graph of f does not have an x-intercept at $x = -5$ because $f(-5) \neq 0$.
(B)	Incorrect. Because the polynomial in the numerator and the polynomial in the denominator of the rational function f have no zeros in common, it can not be concluded that the graph of f has a hole at $x = -5$.
(C)	Correct. The values of the rational function f increase without bound for input values near -5 and greater than -5, and decrease without bound for input values near -5 and less than -5. Based on this information from the table, it is possible that the graph of f has a vertical asymptote at $x = -5$.
(D)	Incorrect. Based on the information from the table, the output values of the rational function f do not get arbitrarily close to -5 as input values increase or decrease without bound. It can not be concluded that the graph of f has a horizontal asymptote at $y = -5$.

Question 84

Skill	Learning Objective	Topic
3.B: Apply numerical results in a given mathematical or applied context.	2.14.A: Construct a logarithmic function model.	2.14: Logarithmic Function Context and Data Modeling

(A)	Incorrect. This is a result of calculating an exponential regression $y = ab^x$ to model the data and evaluating the model for $x = 4.5$. In Desmos, use log mode with the exponential regression to produce the same result as a handheld graphing calculator.
(B)	Incorrect. This is a result of calculating a linear regression to model the data and evaluating the model for $x = 4.5$.
(C)	Incorrect. This is a result of calculating a quadratic regression to model the data and evaluating the model for $x = 4.5$.
(D)	Correct. Using a graphing calculator, a logarithmic regression of the data is calculated as approximately $y = 1167.058 + 2777.948 \ln x$, where x is in years. Store the regression model in the calculator without rounding the values of the coefficients, and use the stored model to evaluate for $x = 4.5$. The model predicts a weight, to the nearest pound, of 5345 pounds at age 4.5 years. In Desmos, use log mode with the logarithmic regression to produce the same result as a handheld graphing calculator.

Question 85

Skill	Learning Objective	Topic
3.A: Describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools.	3.15.A: Describe characteristics of the graph of a polar function.	3.15: Rates of Change in Polar Functions

(A)	Incorrect. This response is the result of thinking that the distance between the point with polar coordinates $\left(f(\theta), \theta\right)$ and the origin is decreasing only when the polar function, $r = f(\theta)$, is positive and decreasing, which occurs on the interval $\left(0, \frac{2\pi}{3}\right) = (0, 2.094)$. This ignores when the polar function is negative and increasing.
(B)	Incorrect. This response is the result of thinking that the distance between the point with polar coordinates $\left(f(\theta), \theta\right)$ and the origin is decreasing only when the polar function, $r = f(\theta)$, is negative, which occurs on the interval $\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right) = (2.094, 4.189)$.
(C)	Correct. Use the graphing calculator to examine the behavior of the graph. The distance between the point with polar coordinates $\left(f(\theta), \theta\right)$ and the origin is decreasing when the polar function, $r = f(\theta)$, is positive and decreasing or negative and increasing. For $f(\theta) = 1 + 2\cos\theta$, this occurs on the interval $\left(0, \frac{2\pi}{3}\right) = (0, 2.094)$, where $r = f(\theta)$ is positive and decreasing, and the interval $\left(\pi, \frac{4\pi}{3}\right) = (3.142, 4.189)$, where $r = f(\theta)$ is negative and increasing.
(D)	Incorrect. This response is the result of thinking that the distance between the point with polar coordinates $\left(f(\theta), \theta\right)$ and the origin is decreasing when the polar function, $r = f(\theta)$, is negative and decreasing, which occurs on the interval $\left(\frac{2\pi}{3}, \pi\right) = (2.094, 3.142)$, and positive and increasing, which occurs on the interval $\left(\frac{4\pi}{3}, 2\pi\right) = (4.189, 6.283)$.

Question 86

Skill	Learning Objective	Topic
3.B: Apply numerical results in a given mathematical or applied context.	3.9.A: Construct analytical and graphical representations of the inverse of the sine, cosine, and tangent functions over a restricted domain.	3.9: Inverse Trigonometric Functions

(A)	Incorrect. This response is the result of finding the first value of t where the graphs of $y = 4.217 \tan\left(0.7t - 0.026\right)$ and $y = 4$ intersect, neglecting that function h involves inverse tangent.
(B)	Correct. Using the graphing calculator, find the value of t where the graphs of $y = h\left(t\right)$ and $y = 4$ intersect. In this case, $y = 4$ represents 4000. The two graphs intersect at the point where $t = 2.029$. This value of t is the time, in hours, when person number 4000 entered the museum.
(C)	Incorrect. This response is the value of $h\left(4\right)$, the number of people, in thousands, that have entered the museum at time $t = 4$.
(D)	Incorrect. This response is the value of $h\left(4000\right)$.

Question 87

Skill	Learning Objective	Topic
2.B: Construct equivalent graphical, numerical, analytical, and verbal representations of functions that are useful in a given mathematical or applied context, with and without technology.	1.1.B: Construct a graph representing two quantities that vary with respect to each other in a contextual scenario.	1.1: Change in Tandem

(A)	Incorrect. This graph meets the requirements for the graph of $y = R(t)$. The rate at which customers arrive at the restaurant is increasing for $0 < t < 4$ and $8 < t < 12$ and decreasing for $4 < t < 8$ and $12 < t < 16$. This could not be the graph of $y = N(t)$.
(B)	Incorrect. The rate at which customers arrive at the restaurant is increasing for $0 < t < 4$ and $8 < t < 12$ and decreasing for $4 < t < 8$ and $12 < t < 16$. This graph has the intervals of "increasing" and intervals of "decreasing" reversed for the graph of $y = R(t)$. This could not be the graph of $y = N(t)$.
(C)	Incorrect. The rate at which customers arrive at the restaurant is increasing for $0 < t < 4$ and $8 < t < 12$ and decreasing for $4 < t < 8$ and $12 < t < 16$. Therefore, the graph of N is concave up on $0 < t < 4$ and $8 < t < 12$. The graph of N is concave down for $4 < t < 8$ and $12 < t < 16$. This graph has the intervals for "concave up" and intervals for "concave down" reversed. This could not be the graph of $y = N(t)$.
(D)	Correct. The rate at which customers arrive at the restaurant is increasing for $0 < t < 4$ and $8 < t < 12$ and decreasing for $4 < t < 8$ and $12 < t < 16$. Therefore, the graph of N is concave up on $0 < t < 4$ and $8 < t < 12$. The graph of N is concave down for $4 < t < 8$ and $12 < t < 16$. This graph meets all of these requirements for the graph of $y = N(t)$. The total number of customers is increasing at an increasing rate on those intervals on which the graph of N is concave up. The total number of customers is increasing at a decreasing rate on those intervals on which the graph of N is concave down.

Answer Key and Questions Alignment

Multiple-Choice Question	Answer	Skill	Learning Objective	Essential Knowledge
1	A	1.A	2.13.A	2.13.A.1
2	B	1.A	2.9.A	2.9.A.1
3	D	1.A	3.3.A	3.3.A.2
4	C	1.B	2.12.A	2.12.A.2
5	D	3.A	2.3.A	2.3.A.3
6	B	1.C	1.10.A	1.10.A.2
7	B	2.A	1.1.B	1.1.B.2
8	C	1.C	2.5.B	2.5.B.1
9	C	1.B	3.12.B	3.12.B.4
10	A	1.B	3.13.A	3.13.A.2
11	A	3.B	2.5.A	2.5.A.3
12	D	3.C	1.5.A	1.5.A.6
13	B	1.C	2.13.A	2.13.A.1
14	B	1.B	1.11.C	1.11.C.1
15	A	2.B	3.6.A	3.6.A.6
16	C	2.A	1.2.A	1.2.A.3
17	D	3.C	3.13.A	3.13.A.1
18	B	1.C	3.6.A	3.6.A.6
19	C	1.A	3.2.A	3.2.A.1
20	B	1.B	2.4.A	2.4.A.1
21	C	3.C	1.3.B	1.3.B.3
22	B	1.C	3.6.A	3.6.A.3
23	A	3.A	2.7.B	2.7.B.1
24	C	2.A	3.2.A	3.2.A.4
25	D	3.A	3.11.A	3.11.A.5
26	C	3.C	1.10.A	1.10.A.1
27	B	3.A	2.6.B	2.6.B.2
28	A	2.A	3.14.A	3.14.A.2
76	A	2.A	1.3.A	1.3.A.3
77	B	3.B	1.14.A	1.14.A.3
78	D	2.A	1.1.B	1.1.B.4
79	C	2.A	3.8.A	3.8.A.1
80	C	1.C	2.15.A	2.15.A.1
81	A	3.B	1.14.C	1.14.C.1
82	D	3.C	1.14.C	1.14.C.1
83	C	2.A	1.9.A	1.9.A.2
84	D	3.B	2.14.A	2.14.A.4
85	C	3.A	3.15.A	3.15.A.2
86	B	3.B	3.9.A	3.9.A.2
87	D	2.B	1.1.B	1.1.B.2