AP® Precalculus Exam

SECTION II: Free Response, Questions

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour
Number of Questions
4
Percent of Total Score
37.5%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
2
Time
30 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
50%

Part B

Number of Questions
2
Time
30 minutes
Electronic Device
None allowed
Percent of Section II Score
50%

Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. You may use the pages in this orange booklet for scratch work, but you must write your answers in the separate Section II: Free Response booklet. **No credit will be given for any work written in this orange booklet**. In the Free Response booklet, write your solution to each part of each question in the space provided for that part. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

- Manage your time carefully. As you begin each part, you may wish to look over both questions for that part before starting to work on them. You are encouraged to use the allotted time to respond to all parts of all questions.
- Show all of your work. Your work will be scored on the correctness and completeness of your responses, including your supporting work and answers. Answers without supporting work may not receive credit in cases where supporting work is requested.
- During Part A, work only on questions 1 and 2. You are expected to use your graphing calculator for tasks such as producing graphs and tables, evaluating functions, solving equations, and performing computations.
- **For Part A, your calculator must be in radian mode**. Avoid rounding intermediate computations on the way to the final result. Unless otherwise specified, any decimal approximations reported in your work should be accurate to three places after the decimal point.
- For Part A, it may be helpful to use your graphing calculator to store information such as computed values for constants, functions you are working with, solutions to equations, and any intermediate values. Computations with the graphing calculator that use the stored information help to maintain as much precision as possible and ensure the desired accuracy in final answers.
- During Part B, questions 3 and 4, no calculator is allowed. Carefully read the instructions provided with the questions. You may continue to work on questions 1 and 2 without the use of a calculator.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

PRECALCULUS

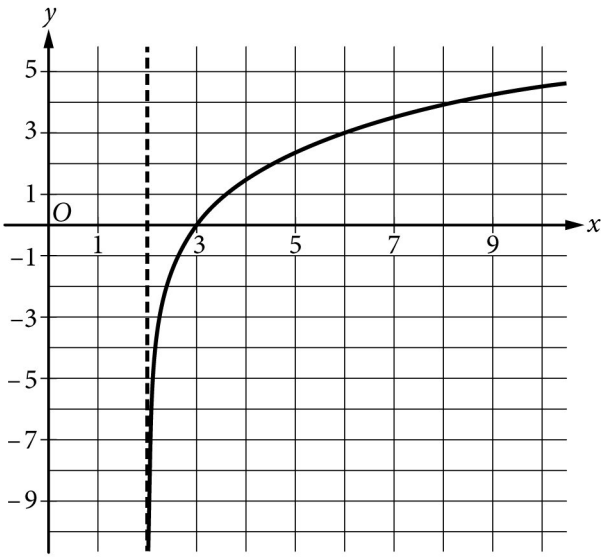
SECTION II, Part A

Time—30 minutes

2 Questions

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

GO ON TO THE NEXT PAGE.



Graph of f

1. The figure shows the graph of the increasing function f on its domain of all real numbers $x > 2$. The points $(3, 0)$ and $(6, 3)$ are on the graph of f. The function g is given by $g(x) = \dfrac{9}{(x-3)}$.

(A) (i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(6)$ as a decimal approximation, or indicate that it is not defined.

(ii) Find all real zeros of f, or indicate there are no real zeros.

(B) (i) Find all values of x, as decimal approximations, for which $g(x) = -5.8$, or indicate that there are no such values.

(ii) Determine the end behavior of g as x decreases without bound. Express your answer using the mathematical notation of a limit.

(C) (i) Determine if f is invertible.

(ii) Give a reason for your answer based on the definition of a function and the graph of $y = f(x)$.

Write your responses to this question only on the designated pages in the separate Free Response booklet.
Write your solution to each part in the space provided for that part.

2 2 2 2 2 2 2 2 2 2

t	0	35	45
Number of Cones	14	57	46

2. The table gives the number of ice cream cones sold by a food vendor. On the initial day ($t = 0$) when the vendor added ice cream cones to the menu, the vendor sold 14 ice cream cones. Thirty-five days later ($t = 35$), the vendor sold 57 ice cream cones. Ten days after that ($t = 45$), the vendor sold 46 ice cream cones.

The number of ice cream cones sold can be modeled by the quadratic function I given by $I(t) = at^2 + bt + c$, where $I(t)$ is the number of ice cream cones sold on day t.

(A) (i) Use the given data to write three equations that can be used to find the values for constants a, b, and c in the expression for $I(t)$.

(ii) Find the values for a, b, and c as decimal approximations.

(B) (i) Use the given data to find the average rate of change of the number of ice cream cones sold, in cones per day, from $t = 35$ to $t = 45$ days. Express your answer as a decimal approximation. Show the computations that lead to your answer.

(ii) Use the average rate of change found in (i) to estimate the number of ice cream cones sold on day $t = 40$. Show the work that leads to your answer.

(iii) Compare the estimate found in (ii) to the value given by the model, $I(40)$. Using characteristics of the average rate of change and characteristics of the quadratic model, explain why the two estimates differ.

(C) Explain how the range values of the function I should be limited by the context of the problem.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

END OF PART A

IF YOU FINISH BEFORE TIME IS CALLED,

YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

PRECALCULUS

SECTION II, Part B

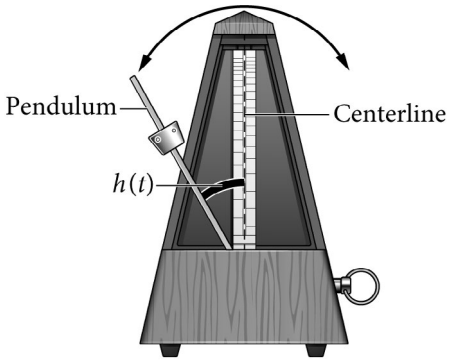
Time—30 minutes

2 Questions

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

NO CALCULATOR ALLOWED



Note: Figure not drawn to scale.

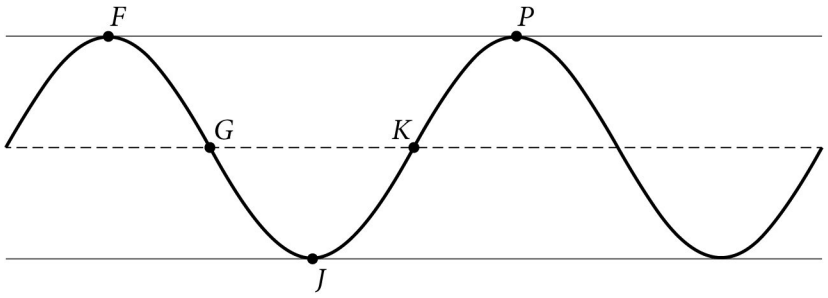
3. A metronome is a device used to help musicians play music at a particular speed. The metronome has a vertical centerline, as shown in the figure. A pendulum on the metronome swings back and forth as it passes the vertical centerline. When the pendulum is farthest to the left or farthest to the right, the measure of the angle formed by the pendulum and the vertical centerline is 0.5 radian.

At time $t = 0$ seconds, the pendulum is farthest to the left. The pendulum then swings to the right and passes the vertical centerline. At time $t = 2$ seconds, the pendulum is farthest to the right for the first time. Then, the pendulum swings left, passes the vertical centerline, and is farthest to the left again at time $t = 4$ seconds. As the pendulum swings, the measure of the angle formed by the pendulum and the vertical centerline periodically increases and decreases.

The sinusoidal function h models the measure of the angle, in radians, formed by the pendulum and the vertical centerline as a function of time t, in seconds. A negative value of $h(t)$ indicates the pendulum is to the left of the vertical centerline; a positive value of $h(t)$ indicates the pendulum is to the right of the vertical centerline.

NO CALCULATOR ALLOWED

(A) The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P, are labeled on the graph. No scale is indicated, and no axes are presented.
Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.



(B) The function h can be written in the form $h(t) = a\cos(b(t+c)) + d$. Find values of constants a, b, c, and d.

(C) Refer to the graph of h in part (A). The t-coordinate of J is t_1, and the t-coordinate of K is t_2.

(i) On the interval (t_1, t_2), which of the following is true about h ?

a. h is positive and increasing.

b. h is positive and decreasing.

c. h is negative and increasing.

d. h is negative and decreasing.

(ii) Describe how the rate of change of h is changing on the interval (t_1, t_2).

Write your responses to this question only on the designated pages in the separate Free Response booklet.

Write your solution to each part in the space provided for that part.

NO CALCULATOR ALLOWED

4. Directions:

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x+3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3+\ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x)=15\arcsin x$$
$$h(x)=\log_{10}(1-x)-\log_{10}4.$$

(i) Solve $g(x)=5\pi$ for values of x in the domain of g.

(ii) Solve $h(x)=1$ for values of x in the domain of h.

(B) The functions j and k are given by

$$j(x)=\log_2(x+4)-11\log_2(x-2)+\log_2\left(x^3\right)$$
$$k(x)=(\cot x)(\csc x).$$

(i) Rewrite $j(x)$ as a single logarithm base 2 without negative exponents in any part of the expression. Your result should be of the form $\log_2(\text{expression})$.

(ii) Rewrite $k(x)$ as a fraction involving powers of $\cos x$ and no other trigonometric functions.

(C) The function m is given by

$$m(x)=\left(2^x\right)^2-3\cdot 2^x.$$

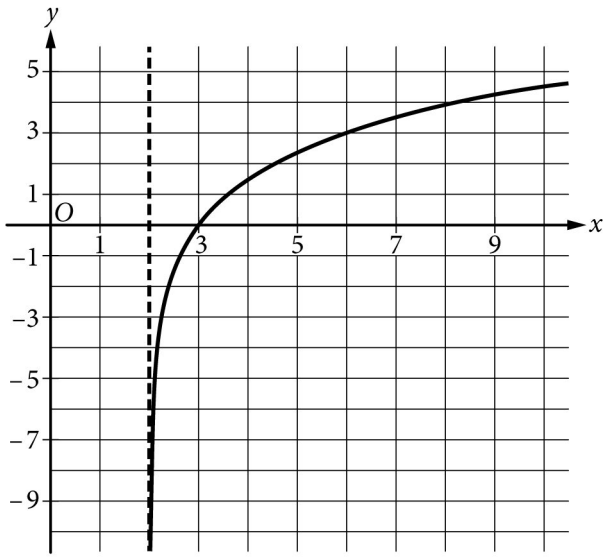
Find all values in the domain of m that yield an output value of 18.

Write your responses to this question only on the designated pages in the separate Free Response booklet.
Write your solution to each part in the space provided for that part.

STOP
END OF EXAM

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

Answer QUESTION 1 part (A) on this page.



Graph of f

Response for question 1(A)

(i)

(ii)

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Answer QUESTION 1 parts (B) and (C) on this page.

Response for question 1(B)

(i)

(ii)

Response for question 1(C)

(i)

(ii)

Answer QUESTION 2 part (A) on this page.

t	0	35	45
Number of Cones	14	57	46

Response for question 2(A)

(i)

(ii)

Answer QUESTION 2 parts (B) and (C) on this page.

Response for question 2(B)

(i)

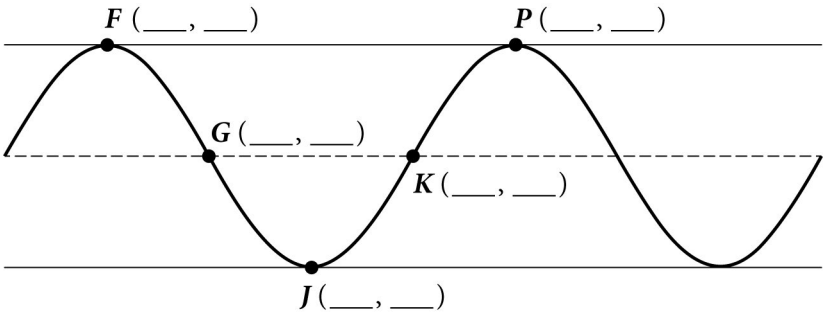
(ii)

(iii)

Response for question 2(C)

Answer QUESTION 3 part (A) on this page.

Response for question 3(A)



Answer QUESTION 3 parts (B) and (C) on this page.

Response for question 3(B)

$a =$ _____

$b =$ _____

$c =$ _____

$d =$ _____

Response for question 3(C)

(i)

(ii)

Answer QUESTION 4 part (A) on this page.

Response for question 4(A)

(i)

(ii)

Answer QUESTION 4 parts (B) and (C) on this page.

Response for question 4(B)

(i)

(ii)

Response for question 4(C)