MCQ_Review_Unit_3

A complex number is represented by a point in the complex plane. The complex number has the rectangular coordinates $(3{,}3)$.

1. Which of the following is one way to express the complex number using its polar coordinates (r, θ) ?

(A) $\left(3\sqrt{2}\cos\left(\frac{\pi}{4}\right)\right) + i\left(3\sqrt{2}\sin\left(\frac{\pi}{4}\right)\right)$

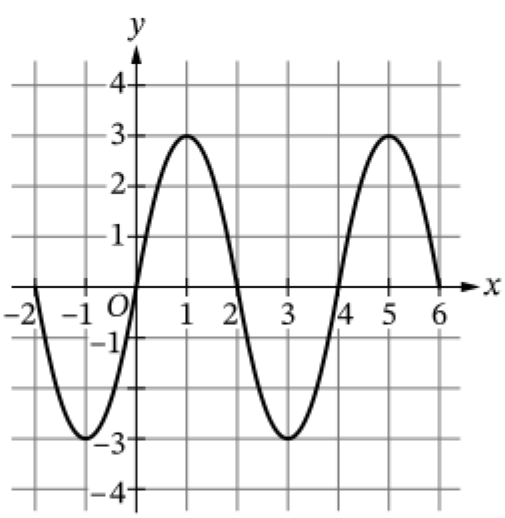
(B) $\left(3\cos\left(\frac{\pi}{4}\right)\right) + i\left(3\sin\left(\frac{\pi}{4}\right)\right)$

(C) $\left(3\sqrt{2}\cos\left(-\frac{\pi}{4}\right)\right) + i\left(3\sqrt{2}\sin\left(-\frac{\pi}{4}\right)\right)$

(D) $\left(3\cos\left(-\frac{\pi}{4}\right)\right) + i\left(3\sin\left(-\frac{\pi}{4}\right)\right)$

Answer A

Correct. The coordinates of $(3{,}3)$ can be converted to polar coordinates (r, θ) and expressed as $(r\cos\theta) + i(r\sin\theta)$ as follows. $r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$ and $\theta = \arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{3}{3}\right) = \frac{\pi}{4}$.



Graph of f

Let f be a sinusoidal function. The graph of $y = f(x)$ is given in the xy-plane.

2. What is the period of f ?

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(A) 2
(B) 3
(C) 4
(D) 6

Answer C

Correct. The period can be determined by the smallest interval of input values over which the maximum or minimum output values start to repeat, that is, the input-value interval between consecutive maxima or consecutive minima. Consecutive maxima occur at $x = 1$ and $x = 5$; consecutive minima occur at $x = -1$ and $x = 3$. The period of f is 4, and $f(x+4) = f(x)$ for all x in the domain.

---

3. The function f is given by $f(x) = \frac{1}{2}\sin x$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. What are the domain and range of the inverse function of f ?

(A) Domain: $\left[-\frac{1}{2}, \frac{1}{2}\right]$, Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
(B) Domain: $[-1,1]$, Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
(C) Domain: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, Range: $\left[-\frac{1}{2}, \frac{1}{2}\right]$
(D) Domain: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, Range: $[-1,1]$

Answer A

Correct. The domain and range of the inverse function are the reverse of the domain and range of the original function. The amplitude of $\frac{1}{2}$ changes the range of the original function. The range of the parent function $y = \sin x$ is $[-1,1]$.

4. Which of the following is the graph of the polar function $r = f(\theta)$, where $f(\theta) = 2\cos(2\theta)$, in the polar coordinate system for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$?

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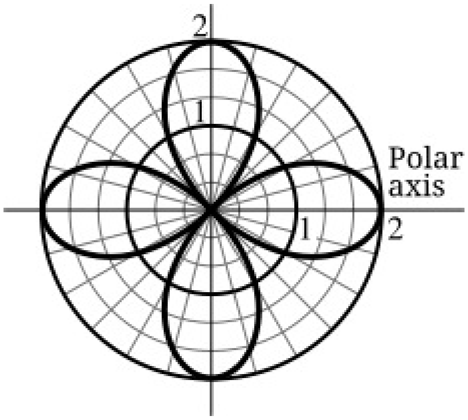
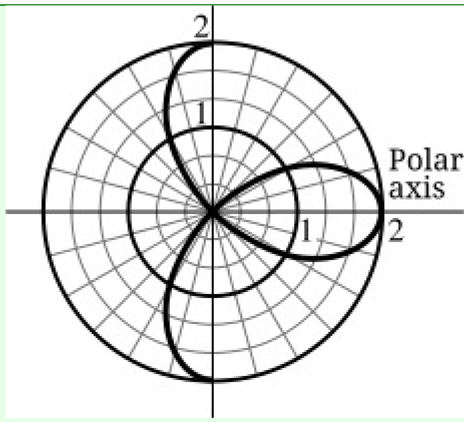
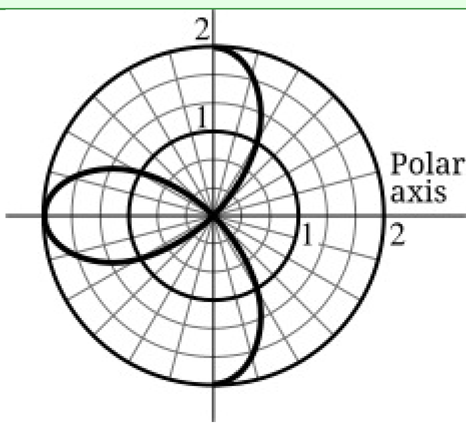
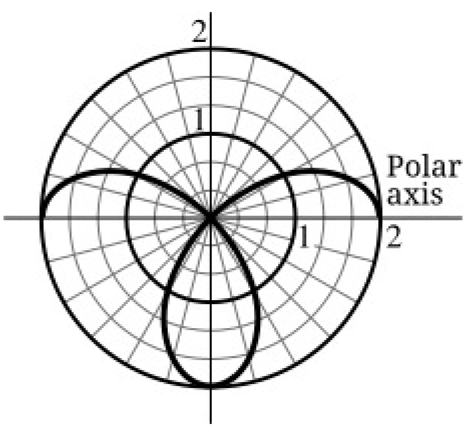
(A) 

(B) 

(C) 

(D) 

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Answer B

Correct. This is the graph of $r = f(\theta)$ for the restricted domain of $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.

5. In the xy-plane, an angle in standard position measures $\frac{5\pi}{6}$ radians. A circle centered at the origin has radius 4. What are the coordinates of the point of intersection of the terminal ray of the angle and the circle?

(A) $\left(-2\sqrt{3}, 2\right)$ ✓

(B) $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

(C) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

(D) $\left(2\sqrt{3}, 2\right)$

Answer A

Correct. The coordinates of the point of intersection are $(r\cos\theta, r\sin\theta)$, where r is the radius of the circle and θ is the angle. The angle is in Quadrant II, where $\sin\theta$ is positive and $\cos\theta$ is negative, and the coordinates are $\left(4 \cdot \cos\left(\frac{5\pi}{6}\right), 4 \cdot \sin\left(\frac{5\pi}{6}\right)\right) = \left(4 \cdot -\frac{\sqrt{3}}{2}, 4 \cdot \frac{1}{2}\right)$.

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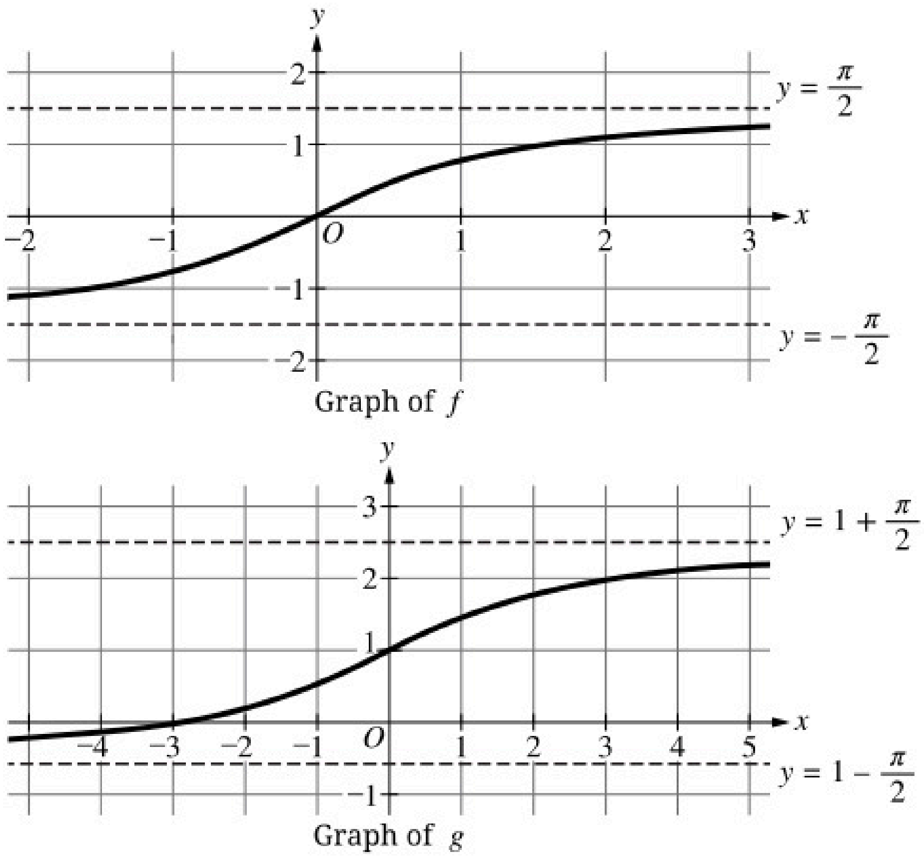
6.



The figure gives the graphs of the functions f and g in the xy-plane. The function f is given by $f(x) = \tan^{-1} x$. Which of the following defines $g(x)$?

(A) $\tan^{-1} x + 1$

(B) $\tan^{-1} x + \frac{\pi}{2}$

(C) $\tan^{-1}\left(\frac{x}{2}\right) + 1$

(D) $\tan^{-1}\left(\frac{x}{2}\right) + \frac{\pi}{2}$

Answer C

Correct. The graph of g is the result of a horizontal dilation of the graph of f and a vertical translation of the graph of f by 1 unit.

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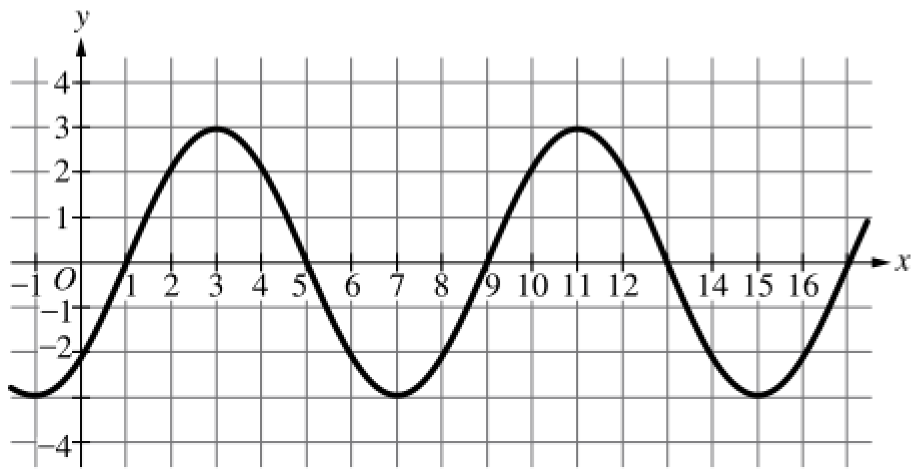
7.



Graph of g

The figure shows the graph of a sinusoidal function g. What are the values of the period and amplitude of g ?

(A) The period is 4, and the amplitude is 3.

(B) The period is 8, and the amplitude is 3.

(C) The period is 4, and the amplitude is 6.

(D) The period is 8, and the amplitude is 6.

Answer B

Correct. From the graph, consecutive maxima of sinusoidal function g occur at $x = 3$ and $x = 11$. Consecutive minima of g occur at $x = -1$ and $x = 7$. Therefore, the period is 8. The maximum value of g is 3, and the minimum value is -3. Therefore, the amplitude is $\frac{3-(-3)}{2} = 3$. (Note: Possible expressions for this function are $g(x) = 3\sin\left(\frac{\pi}{4}(x-1)\right)$ and $g(x) = -3\cos\left(\frac{\pi}{4}(x+1)\right)$.)

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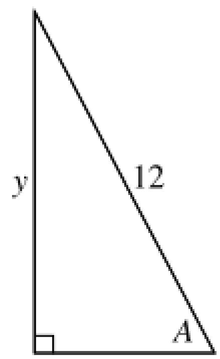
8.



The figure gives a right triangle, where y is the length of the side opposite angle A. If the function f gives values of A as a function of y, which of the following could define $f(y)$?

(A) $f(y) = \cos\left(\frac{y}{12}\right)$

(B) $f(y) = \sin\left(\frac{y}{12}\right)$

(C) $f(y) = \cos^{-1}\left(\frac{y}{12}\right)$

(D) $f(y) = \sin^{-1}\left(\frac{y}{12}\right)$

Answer D

Correct. Using the right triangle definition, $\sin A = \frac{y}{12}$. Therefore, $A = \sin^{-1}\left(\frac{y}{12}\right)$ and $A = f(y)$. The inverse sine function gives an angle measure with the appropriate ratio as the input value.

9. Consider the graph of the polar function $r = f(\theta)$, where $f(\theta) = 4\sin(2\theta)$, in the polar coordinate system. On the interval $0 \leq \theta \leq 2\pi$, which of the following is true about the graph of $r = f(\theta)$?

(A) For the input values $\theta = \frac{\pi}{4}$, $\theta = \frac{3\pi}{4}$, $\theta = \frac{5\pi}{4}$, and $\theta = \frac{7\pi}{4}$, the function $r = f(\theta)$ has extrema that correspond to points that are farthest from the origin.

(B) For the input values $\theta = \frac{\pi}{4}$, $\theta = \frac{3\pi}{4}$, $\theta = \frac{5\pi}{4}$, and $\theta = \frac{7\pi}{4}$, the function $r = f(\theta)$ has extrema. However, only the points corresponding to $\theta = \frac{\pi}{4}$ and $\theta = \frac{5\pi}{4}$ are farthest from the origin.

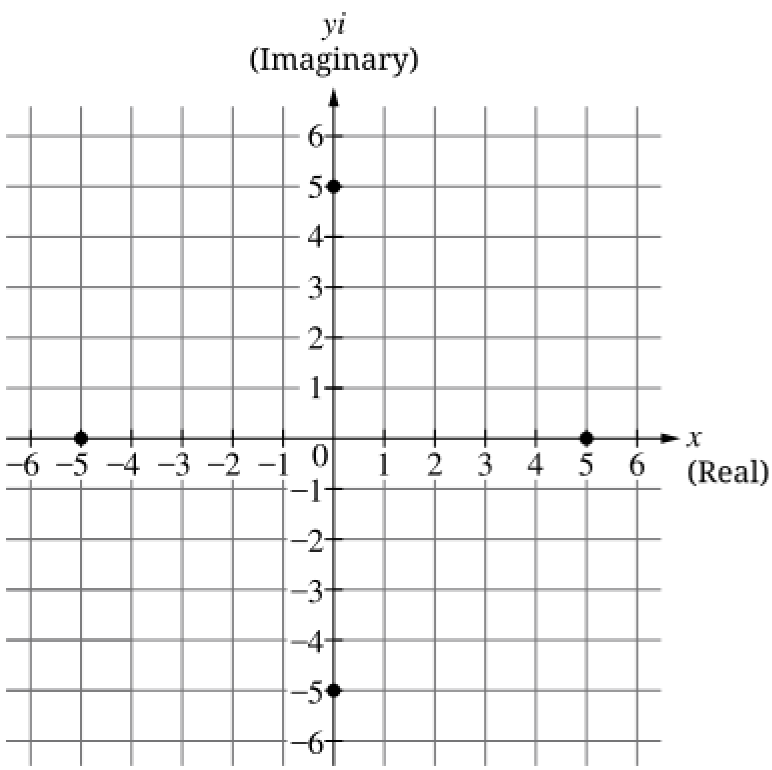
(C) For the input values $\theta = \frac{\pi}{2}$ and $\theta = \frac{3\pi}{2}$, the function $r = f(\theta)$ has extrema that correspond to points that are farthest from the origin.

(D) For the input values $\theta = \frac{\pi}{2}$ and $\theta = \frac{3\pi}{2}$, the function $r = f(\theta)$ has extrema. However, only the point corresponding to $\theta = \frac{\pi}{2}$ is farthest from the origin.

Answer A

Correct. At each of these four angles θ, the values of r are extrema of -4 or 4, and the corresponding distances to the origin are all 4.

10.



The graph indicates four points in the complex plane. Each complex number has polar coordinates (r, θ). Which of the following completes the expression for the four points in the polar form $(5\cos\theta) + i(5\sin\theta)$?

(A) $\theta = \frac{\pi}{4} + \left(\frac{\pi}{4}\right)k$, where $k = 1,\ 2,\ 3,\ 4$

(B) $\theta = \frac{\pi}{4} + \left(\frac{\pi}{2}\right)k$, where $k = 1,\ 2,\ 3,\ 4$

(C) $\theta = \frac{\pi}{2} + \left(\frac{\pi}{2}\right)k$, where $k = 1,\ 2,\ 3,\ 4$

(D) $\theta = \frac{\pi}{2} + \pi k$, where $k = 1,\ 2,\ 3,\ 4$

Answer C

Correct. The four points have rectangular coordinates $(-5{,}0)$, $(0, -5)$, $(5{,}0)$, and $(0{,}5)$. These four

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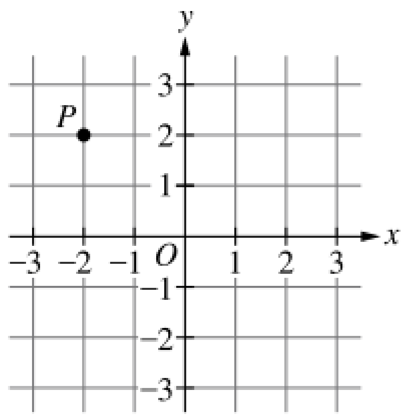
values for θ yield complex numbers -5, $-5i$, 5, and $5i$.

11.



The graph of point P is given in the xy-plane. Which of the following are possible polar coordinates of point P ?

(A) $\left(2, \frac{\pi}{4}\right)$

(B) $\left(2, \frac{3\pi}{4}\right)$

(C) $\left(2\sqrt{2}, \frac{\pi}{4}\right)$

(D) $\left(2\sqrt{2}, \frac{3\pi}{4}\right)$

Answer D

Correct. The rectangular coordinates of P are $(-2,2)$. To convert to polar coordinates, find the value of r: $r = \sqrt{x^2 + y^2} = \sqrt{4+4} = 2\sqrt{2}$. Find the value of θ: $\theta = \arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{2}{-2}\right) = \arctan(-1) = -\frac{\pi}{4}$. Because P is in Quadrant II and $x < 0$, the angle θ is $-\frac{\pi}{4} + \pi = \frac{3\pi}{4}$.

12. The function g is given by $g(\theta) = \cos(2\theta)$. The sinusoidal function h is a phase shift of the function g by $-\frac{\pi}{3}$ units. Which of the following is true?

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(A) Consecutive minima of h occur at $\left(-\frac{5\pi}{6}, h\left(-\frac{5\pi}{6}\right)\right)$ and $\left(\frac{\pi}{6}, h\left(\frac{\pi}{6}\right)\right)$ because consecutive minima of g occur at $\left(-\frac{\pi}{2}, g\left(-\frac{\pi}{2}\right)\right)$ and $\left(\frac{\pi}{2}, g\left(\frac{\pi}{2}\right)\right)$, and h is the image of g with a horizontal shift of $\frac{\pi}{3}$ units left.

(B) Consecutive minima of h occur at $\left(-\frac{\pi}{6}, h\left(-\frac{\pi}{6}\right)\right)$ and $\left(\frac{5\pi}{6}, h\left(\frac{5\pi}{6}\right)\right)$ because consecutive minima of g occur at $\left(-\frac{\pi}{2}, g\left(-\frac{\pi}{2}\right)\right)$ and $\left(\frac{\pi}{2}, g\left(\frac{\pi}{2}\right)\right)$, and h is the image of g with a horizontal shift of $\frac{\pi}{3}$ units right.

(C) Consecutive minima of h occur at $\left(-\frac{4\pi}{3}, h\left(-\frac{4\pi}{3}\right)\right)$ and $\left(\frac{2\pi}{3}, h\left(\frac{2\pi}{3}\right)\right)$ because consecutive minima of g occur at $(-\pi, g(-\pi))$ and $(\pi, g(\pi))$, and h is the image of g with a horizontal shift of $\frac{\pi}{3}$ units left.

(D) Consecutive minima of h occur at $\left(-\frac{2\pi}{3}, h\left(-\frac{2\pi}{3}\right)\right)$ and $\left(\frac{4\pi}{3}, h\left(\frac{4\pi}{3}\right)\right)$ because consecutive minima of g occur at $(-\pi, g(-\pi))$ and $(\pi, g(\pi))$, and h is the image of g with a horizontal shift of $\frac{\pi}{3}$ units right.

Answer A

Correct. The period of g is π, and g is a transformation of $y = \cos\theta$, so the minima of g are correct. The phase shift indicated for h results in the values indicated. The reasoning is correct.

13. In the polar coordinate system, the point A has polar coordinates $\left(5, \frac{\pi}{4}\right)$. Which of the following also gives the location of point A in polar coordinates?

(A) $\left(-5, \frac{3\pi}{4}\right)$

(B) $\left(-5, \frac{5\pi}{4}\right)$

(C) $\left(-5, \frac{7\pi}{4}\right)$

(D) $\left(-5, \frac{9\pi}{4}\right)$

Answer B

Correct. $-r$ is the same distance from the origin as r is, but it is in the direction opposite the terminal ray of the angle θ. A different representation of the point A can be given by $-r$ and an angle θ plus an odd multiple of π. In this case, the angle is $\theta = \frac{\pi}{4} + \pi$.

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14.

x	0	1	2	3	4
y	5	4	3	4	5

The table gives ordered pairs for five points from a larger data set. The larger data set can be modeled by a sinusoidal function f with a period of 4. The maximum values of the data set occur at x-values that are multiples of 4. Which of the following best defines $f(x)$ for the larger data set?

(A) $\cos\left(\frac{\pi}{2}x\right) + 4$

(B) $\cos(\pi x) + 4$

(C) $2\cos\left(\frac{\pi}{2}x\right) + 4$

(D) $2\cos(\pi x) + 4$

Answer A

Correct. The period of the sinusoidal function model is 4. If b is the coefficient of x, then $\frac{2\pi}{b} = 4$ so $b = \frac{\pi}{2}$. The amplitude is $\frac{5-3}{2} = 1$, and the vertical shift is 4.

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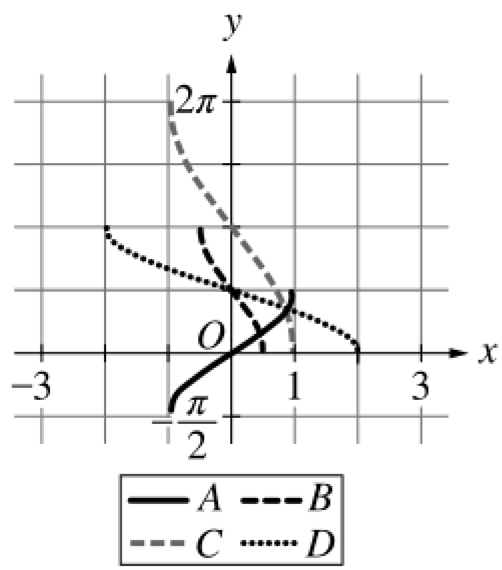
15.



The figure gives the graphs of four functions labeled A, B, C, and D in the xy-plane. Which is the graph of $f(x) = 2\cos^{-1} x$?

(A) A

(B) B

(C) C

(D) D

Answer C

Correct. This is the graph of $f(x) = 2\cos^{-1} x$, which is a vertical dilation of the graph of $y = \cos^{-1} x$ by a factor of 2.

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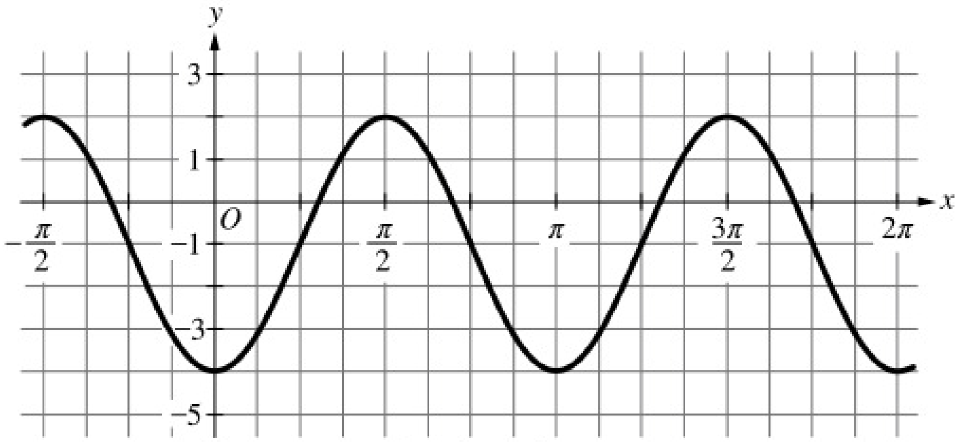

Graph of f

The figure shows the graph of a trigonometric function f.

16. Which of the following could be an expression for $f(x)$?

(A) $3\cos\left(2\left(x-\frac{\pi}{4}\right)\right)-1$

(B) $3\cos\left(2\left(x-\frac{\pi}{8}\right)\right)-1$

(C) $3\sin\left(2\left(x-\frac{\pi}{4}\right)\right)-1$

(D) $3\sin\left(2\left(x-\frac{\pi}{8}\right)\right)-1$

Answer C

Correct. The amplitude of the function is $\frac{2-(-4)}{2}=3$, so $a=3$. The period is the input-value interval between consecutive maxima or consecutive minima, and is $\frac{3\pi}{2}-\frac{\pi}{2}=\pi$. Therefore, $\pi=\frac{2\pi}{b}\rightarrow b=2$. The midline is $y=-1$, so $d=-1$. Comparing the behavior of this graph to the graph of $y=\sin x$, this graph is translated right with a phase shift of $\frac{\pi}{4}$ units. Therefore, $c=-\frac{\pi}{4}$. Putting this together in the form of $y=a\sin(b(x+c))+d$ results in $f(x)=3\sin\left(2\left(x-\frac{\pi}{4}\right)\right)-1$. As a cosine function, this could be written as $f(x)=-3\cos(2x)-1$, where the phase shift is 0 and the -3 indicates a reflection over the x-axis.

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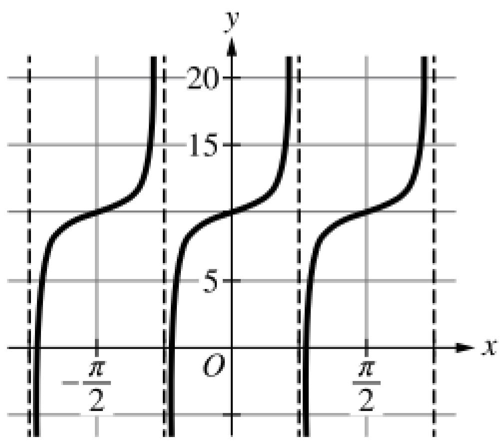
17.



Graph of h

The graph of the function h is given in the xy-plane. If $h(x) = a\tan(bx) + 10$, where a and b are constants, which of the following is true?

(A) $a > 0$ and $b > 1$

(B) $a > 0$ and $0 < b < 1$

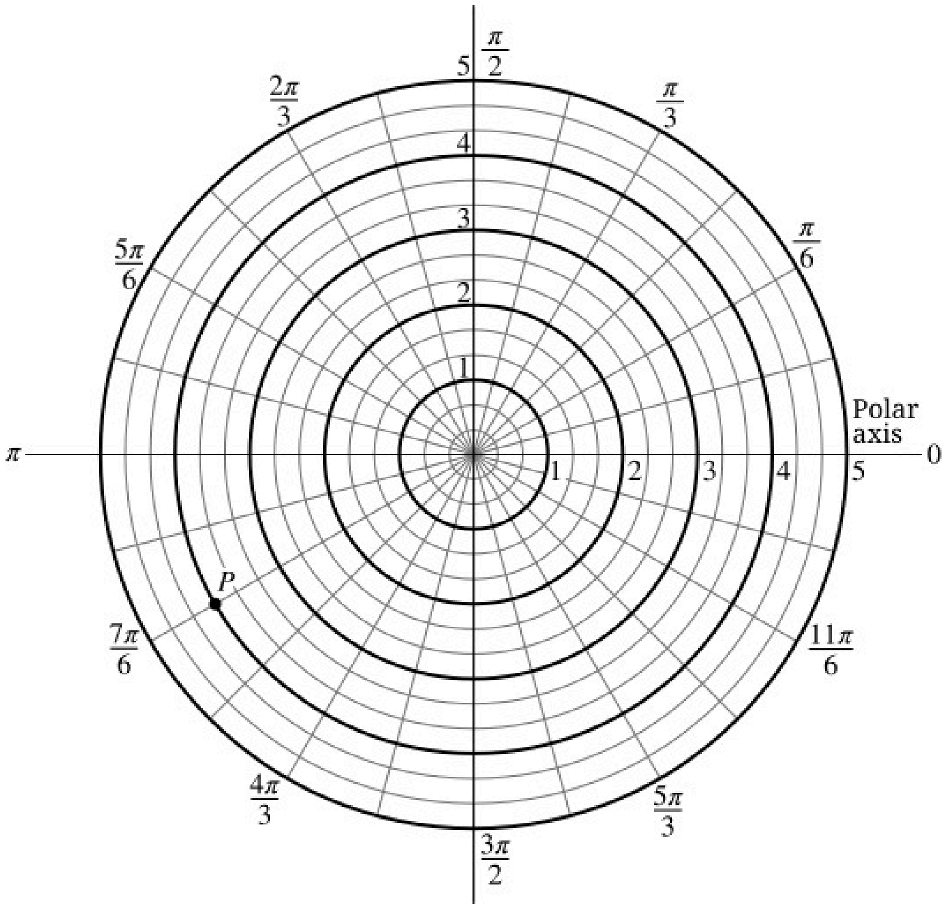
(C) $a < 0$ and $b > 1$

(D) $a < 0$ and $0 < b < 1$

Answer A

Correct. The value of a is greater than 0 because function h does not involve a reflection of the graph of the tangent function over a horizonal line. The period of the function is less than π, the period of $y = \tan x$, so $\frac{\pi}{b} < \pi$ implies $b > 1$.

18.



The figure shows the polar coordinate system with point P labeled. Point P is rotated an angle of measure $\frac{\pi}{2}$ clockwise about the origin. The image of this transformation is at the location K (not shown). What are the rectangular coordinates of K ?

(A) $\left(-2,2\sqrt{3}\right)$

(B) $\left(-2\sqrt{3},2\right)$

(C) $\left(2,-2\sqrt{3}\right)$

(D) $\left(2\sqrt{3},-2\right)$

Answer A

Correct. The location of P is $\left(4,\frac{7\pi}{6}\right)$. Therefore, the location of K is $\left(4,\frac{7\pi}{6}-\frac{\pi}{2}\right)=\left(4,\frac{2\pi}{3}\right)$. Converting to rectangular coordinates (x,y) yields $(r\cos\theta,r\sin\theta)=\left(4\cos\left(\frac{2\pi}{3}\right),4\sin\left(\frac{2\pi}{3}\right)\right)$.

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Therefore, the rectangular coordinates are $\left(4 \cdot -\frac{1}{2}, 4 \cdot \frac{\sqrt{3}}{2}\right)$.

19. Which of the following expresses the complex number $10 + 10i$ using polar coordinates in the form $(r \cos \theta) + i(r \sin \theta)$?

(A) $\left(\frac{1}{10} \cos\left(\frac{\pi}{4}\right)\right) + i\left(\frac{1}{10} \sin\left(\frac{\pi}{4}\right)\right)$

(B) $(10 \cos 0) + i\left(10 \sin\left(\frac{\pi}{2}\right)\right)$

(C) $\left(10 \cos\left(\frac{\pi}{4}\right)\right) + i\left(10 \sin\left(\frac{\pi}{4}\right)\right)$

(D) $\left(10\sqrt{2} \cos\left(\frac{\pi}{4}\right)\right) + i\left(10\sqrt{2} \sin\left(\frac{\pi}{4}\right)\right)$ ✓

Answer D

Correct. The rectangular coordinates of the complex number in the complex plane are $(10,10)$. The polar coordinates of the complex number are $(r, \theta) = \left(\sqrt{x^2 + y^2}, \arctan\left(\frac{y}{x}\right)\right)$, where $x > 0$ in this case. Therefore, $r = \sqrt{10^2 + 10^2} = \sqrt{200} = 10\sqrt{2}$ and $\theta = \arctan\left(\frac{10}{10}\right) = \arctan 1 = \frac{\pi}{4}$.

20. Which of the following is the graph of $f(x) = 2 \csc\left(\frac{2\pi}{3} x\right)$ in the xy-plane?

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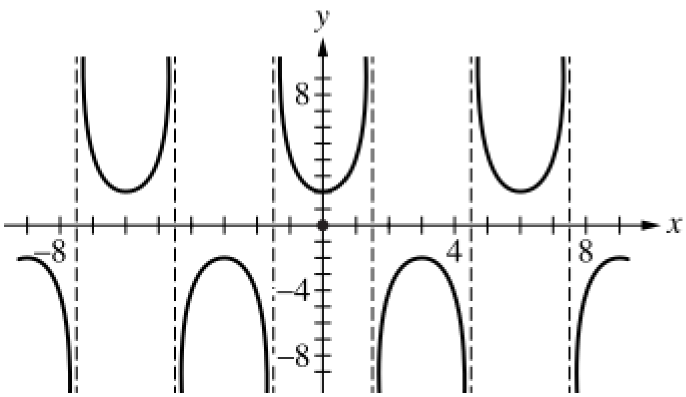
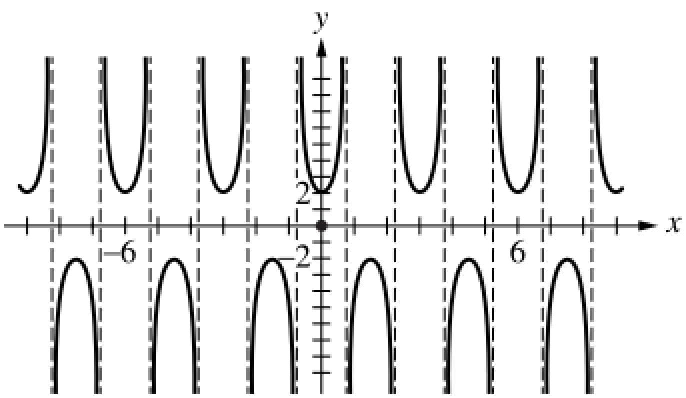
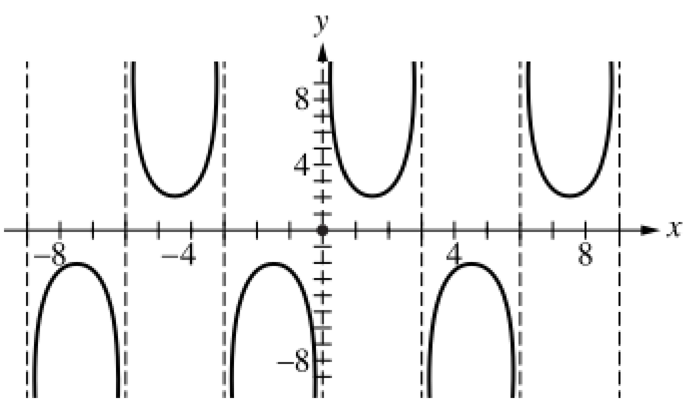
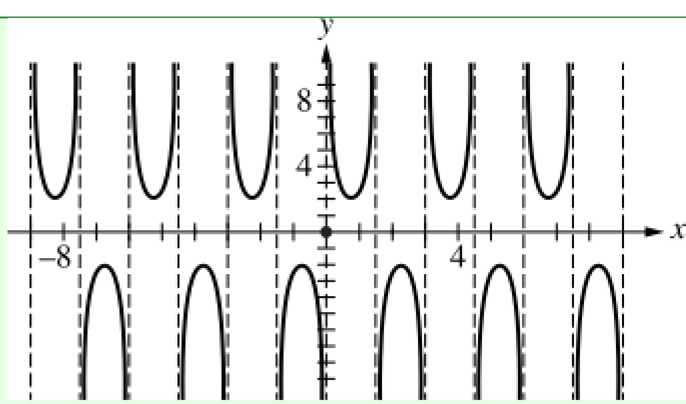
(A)


(B)


(C)


(D)


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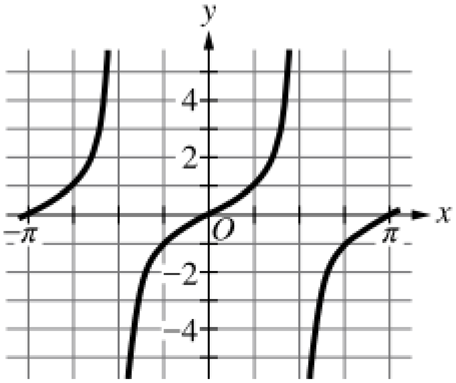
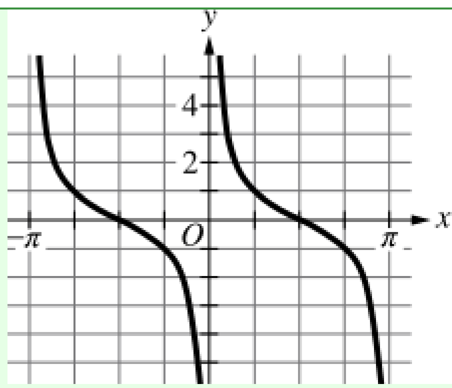
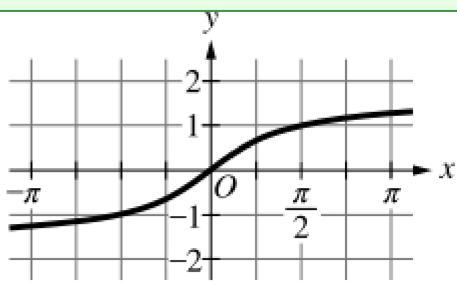
Answer D

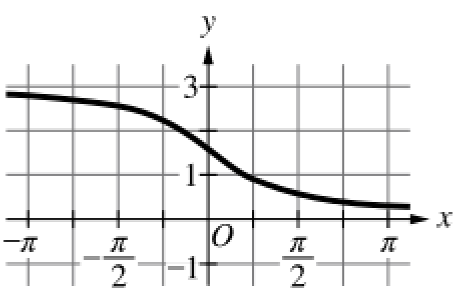
Correct. Cosecant is the reciprocal of the sine function, and the graph of cosecant has vertical asymptotes where the value of sine is 0. The period is $\frac{2\pi}{(2\pi/3)} = 3$. In this case, the vertical asymptotes occur at $x = 1.5k$, where k is any integer. The range of this function is $(-\infty, -2] \cup [2, \infty)$.

21. Which of the following is the graph of $f(x) = \cot x$ in the xy-plane?

(A)


(B)


(C)


(D)


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Answer B

Correct. This is the graph of the cotangent function, which is the reciprocal of the tangent function where $\tan x \neq 0$. The vertical asymptotes are at values of x where $\tan x = 0$, and the function is decreasing between consecutive asymptotes.

22. The function f is given by $f(t) = \sin^2 t - 1$. For how many values of t does $f(t) = 0$?

(A) None

(B) One

(C) Two

(D) Infinitely many ✓

Answer D

Correct. f is a periodic function with domain of all real numbers and a period of π. $f(t) = 0$ for odd multiples of $\frac{\pi}{2}$. Thus, there are infinitely many solutions because the function does not have any domain restrictions.

23. In the xy-plane, the terminal ray of angle θ in standard position intersects a circle of radius r at the point $\left(10, -10\sqrt{3}\right)$. What are the values of θ and r ?

(A) $\theta = \frac{5\pi}{3}$ and $r = 10$

(B) $\theta = \frac{5\pi}{3}$ and $r = 20$ ✓

(C) $\theta = \frac{11\pi}{6}$ and $r = 10$

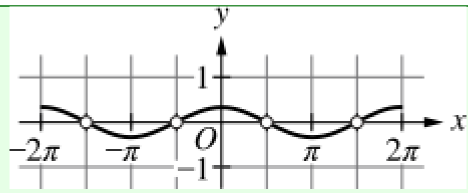
(D) $\theta = \frac{11\pi}{6}$ and $r = 20$

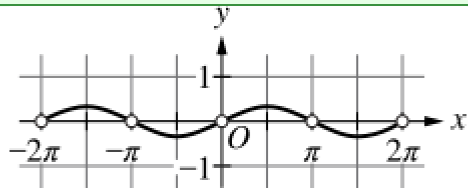
Answer B

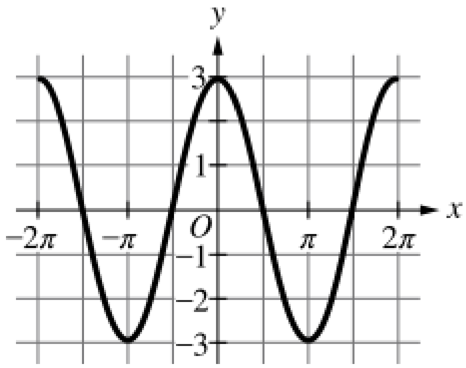
Correct. Based on the information given, $r\cos\theta = 10$ and $r\sin\theta = -10\sqrt{3}$. Because of the signs of the values, the terminal ray of the angle is in Quadrant IV. Considering multiples of $\frac{\pi}{6}$ in this quadrant, $\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}$ and $\sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$. Multiplying these values by 20 gives our desired coordinates.

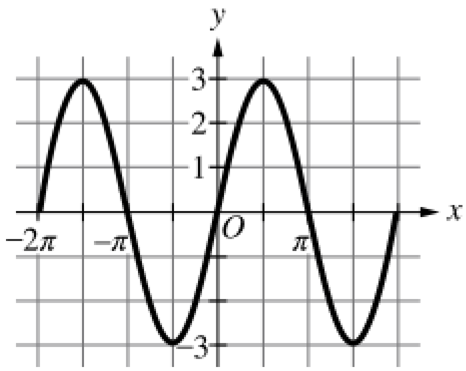
MCQ_Review_Unit_3

24. The function f is defined by $f(x) = 3\sec x$. The function g is defined by $g(x) = \frac{1}{f(x)}$. Which of the following is the graph of g in the xy-plane?

(A) 

(B) 

(C) 

(D) 

Answer A

Correct. $g(x)$ is the reciprocal of $f(x)$. Because $f(x) = 3\sec x,\ g(x) = \frac{1}{3}\cos x$. $f(x)$ is not defined where cosine is equal to 0. This occurs at $x = \frac{\pi}{2} + \pi k$, where k is any integer, so the graph of g has holes at those values.

MCQ_Review_Unit_3

25. The function g is given by $g(x) = 3\csc(\pi(x+2)) - 1$. Which of the following describes the range of g ?

(A) The range of g is $[-3,3]$.

(B) The range of g is $(-\infty, -2] \cup [4, \infty)$.

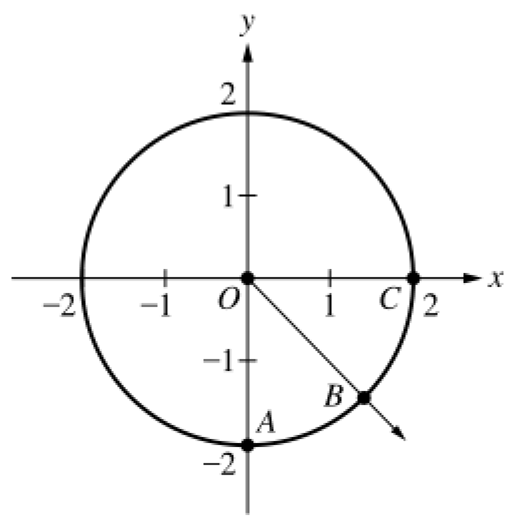
(C) The range of g is $(-\infty, -3] \cup [3, \infty)$.

(D) The range of g is $(-\infty, -4] \cup [2, \infty)$.

Answer D

Correct. This is the result of appropriately accounting for the vertical dilation and vertical translation of the graph of the cosecant function.

26.



The figure shows a circle of radius 2 along with four labeled points in the xy-plane. The measure of angle COB is equal to the measure of angle AOB. What are the coordinates of point B ?

(A) $\left(\cos\left(\frac{7\pi}{4}\right), \sin\left(\frac{7\pi}{4}\right)\right)$

(B) $\left(\sin\left(\frac{7\pi}{4}\right), \cos\left(\frac{7\pi}{4}\right)\right)$

(C) $\left(2\cos\left(\frac{7\pi}{4}\right), 2\sin\left(\frac{7\pi}{4}\right)\right)$

(D) $\left(2\sin\left(\frac{7\pi}{4}\right), 2\cos\left(\frac{7\pi}{4}\right)\right)$

MCQ_Review_Unit_3

Answer C

Correct. Because the measure of angle COB is equal to the measure of angle AOB, ray OB is at an angle of $\frac{7\pi}{4}$. This represents $\frac{3}{4}$ of the circle. With a circle of radius 2, these are the coordinates of point B.

27. Which of the following describes the graph of $f(x) = \cot x$?

(A) The graph has vertical asymptotes at $x = \frac{\pi}{2} + \pi k$, where k is any integer, and the function is increasing on all intervals in its domain.

(B) The graph has vertical asymptotes at $x = \frac{\pi}{2} + \pi k$, where k is any integer, and the function is decreasing on all intervals in its domain.

(C) The graph has vertical asymptotes at $x = \pi + \pi k$, where k is any integer, and the function is increasing on all intervals in its domain.

(D) The graph has vertical asymptotes at $x = \pi + \pi k$, where k is any integer, and the function is decreasing on all intervals in its domain.

Answer D

Correct. The graph has vertical asymptotes for values of x where $\sin x = 0$. The function is decreasing between consecutive asymptotes on its domain.

28. Angles A and B are in standard position in the xy-plane. The measure of angle A is $\frac{2\pi}{3}$ radians, and the measure of angle B is $\frac{4\pi}{3}$ radians. The terminal rays of both angles intersect a circle centered at the origin with radius 20. What is the distance between these two points of intersection: the circle and terminal ray of angle A, and the circle and terminal ray of angle B ?

(A) $\cos\left(\frac{2\pi}{3}\right) - \cos\left(\frac{4\pi}{3}\right)$

(B) $20\cos\left(\frac{2\pi}{3}\right) - 20\cos\left(\frac{4\pi}{3}\right)$

(C) $\sin\left(\frac{2\pi}{3}\right) - \sin\left(\frac{4\pi}{3}\right)$

(D) $20\sin\left(\frac{2\pi}{3}\right) - 20\sin\left(\frac{4\pi}{3}\right)$

Answer D

Correct. Because the points of intersection are reflections of each other over the x-axis, the y-coordinates of the points can be used to calculate the distance.

29. The function f is defined by $f(x) = \sec\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right)$. Which of the following describes the domain of f ?

(A) The domain is the set of all real numbers x, except when $x = \frac{\pi}{2} + \pi k$, where k is any integer.

(B) The domain is the set of all real numbers x, except when $x = \frac{\pi}{2} + 2\pi k$, where k is any integer.

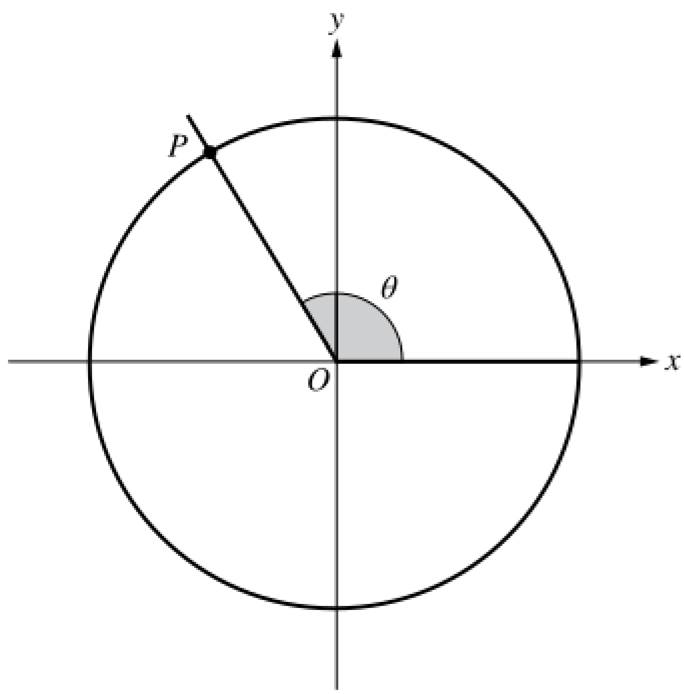
(C) The domain is the set of all real numbers x, except when $x = \pi + 2\pi k$, where k is any integer.

(D) The domain is the set of all real numbers x, except when $x = \frac{3\pi}{2} + 2\pi k$, where k is any integer.

Answer D

Correct. The secant function is not defined where the cosine function is equal to 0. This is the result of finding where $\cos\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right) = 0$ to determine the excluded values of the domain.

30.



The figure shows the terminal ray of angle θ, in standard position, intersecting the unit circle at point P in the xy-plane. The function g is given by $g(z) = \cos z$. For the angle ω (not shown), $\theta < \omega < \pi$. Which of the following is true?

(A) $g(\omega) < g(\theta)$

(B) $g(\omega) > g(\theta)$

(C) $g(\omega) = g(\theta)$

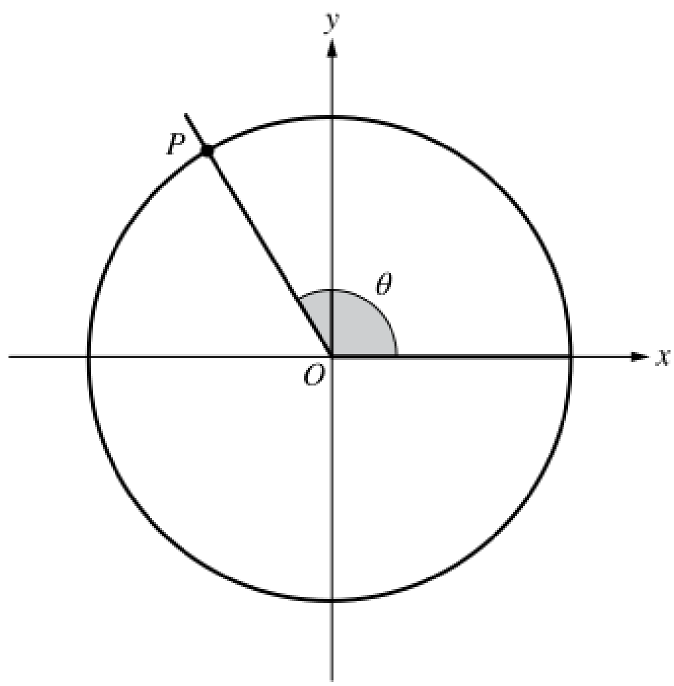
(D) Depending on the value of ω, sometimes $g(\omega) < g(\theta)$ and sometimes $g(\omega) > g(\theta)$.

MCQ_Review_Unit_3

Answer A

Correct. Because this is the unit circle with radius 1, the cosine function gives the horizontal displacement of P from the y-axis. Given that $\theta < \omega < \pi$, the x-coordinates, or values of $g(\omega)$, are less than the x-coordinate of P, $g(\theta)$.

31.



The figure shows the terminal ray of angle θ, in standard position, intersecting the unit circle at point P in the xy-plane. The function f is given by $f(z) = \sin z$. For the angle β (not shown), $\frac{\pi}{2} < \beta < \theta$. Which of the following is true?

(A) $f(\beta) < f(\theta)$

(B) $f(\beta) > f(\theta)$

(C) $f(\beta) = f(\theta)$

(D) Depending on the value of β, sometimes $f(\beta) < f(\theta)$ and sometimes $f(\beta) > f(\theta)$.

MCQ_Review_Unit_3

Answer B

Correct. Because this is the unit circle with radius 1, the sine function gives the vertical displacement of P from the x-axis. Given that $\frac{\pi}{2} < \beta < \theta$, the y-coordinates, or values of $f(\beta)$, are greater than the y -coordinate of P, $f(\theta)$.

32. The function g is given by $g(\theta) = \cos\theta$. Which of the following describes g on the interval from $\theta = \frac{3\pi}{2}$ to $\theta = 2\pi$?

(A) g is decreasing, and the graph of g is concave down.

(B) g is decreasing, and the graph of g is concave up.

(C) g is increasing, and the graph of g is concave down.

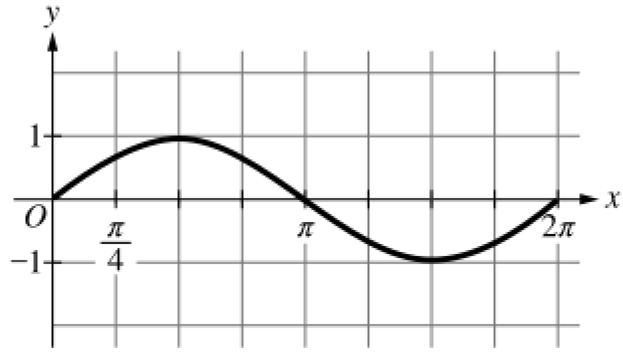
(D) g is increasing, and the graph of g is concave up.

Answer C

Correct. The function is increasing on this interval. The rate of change is decreasing on this interval. Therefore, the graph of g is concave down on this interval. Another way to think of this is as follows. In the xy-plane, let P be the point of intersection of the terminal ray of an angle θ in standard position and the unit circle. As θ increases on the interval $\frac{3\pi}{2} < \theta < 2\pi$, the x-coordinates of P are increasing at a decreasing rate. The behavior of the x-coordinates of P on the unit circle corresponds to the behavior of the cosine function.

MCQ_Review_Unit_3

33.



Graph of f

The figure shows the graph of the function f in the xy-plane. The function f is either a sine function or a cosine function. Consider an angle in standard position in another xy-plane (not shown). The terminal ray of the angle intersects the unit circle at point P. Which of the following is true about the relationship between f and P ?

(A) f gives the vertical distance of P from the x-axis for angle measures from 0 to 2π.

(B) f gives the horizontal distance of P from the y-axis for angle measures from 0 to 2π.

(C) f gives the vertical displacement of P from the x-axis for angle measures from 0 to 2π.

(D) f gives the horizontal displacement of P from the y-axis for angle measures from 0 to 2π.

Answer C

Correct. Because the graph of f begins at the origin, increases to a maximum, decreases to a zero at π, decreases to a minimum, then increases to a zero again at 2π, the graph is the sine function $y = \sin x$. This gives the vertical displacement of P from the x-axis. (The radius of the unit circle is 1.)

34. The function f is given by $f(\theta) = \cos\theta$, and the function g is given by $g(\theta) = \sin\theta$. Which of the following describes f and g on the interval from $\theta = \frac{\pi}{2}$ to $\theta = \pi$?

(A) Both f and g are decreasing.

(B) Both f and g are increasing.

(C) f is decreasing, and g is increasing.

(D) f is increasing, and g is decreasing.

MCQ_Review_Unit_3

Answer A

Correct. Both functions are decreasing on this interval. In the xy-plane, let P be the point of intersection of the terminal ray of an angle θ in standard position and the unit circle. As θ increases on the interval $\frac{\pi}{2} < \theta < \pi$, the x-coordinates of P are decreasing, and the y-coordinates of P are decreasing. The behavior of the x-coordinates of P on the unit circle corresponds to the behavior of the cosine function. The behavior of the y-coordinates of P on the unit circle corresponds to the behavior of the sine function.

35. The function f is given by $f(\theta) = \sin\theta$. Which of the following describes f on the interval from $\theta = \pi$ to $\theta = \frac{3\pi}{2}$?

(A) f is decreasing, and the graph of f is concave down.

(B) f is decreasing, and the graph of f is concave up. ✓

(C) f is increasing, and the graph of f is concave down.

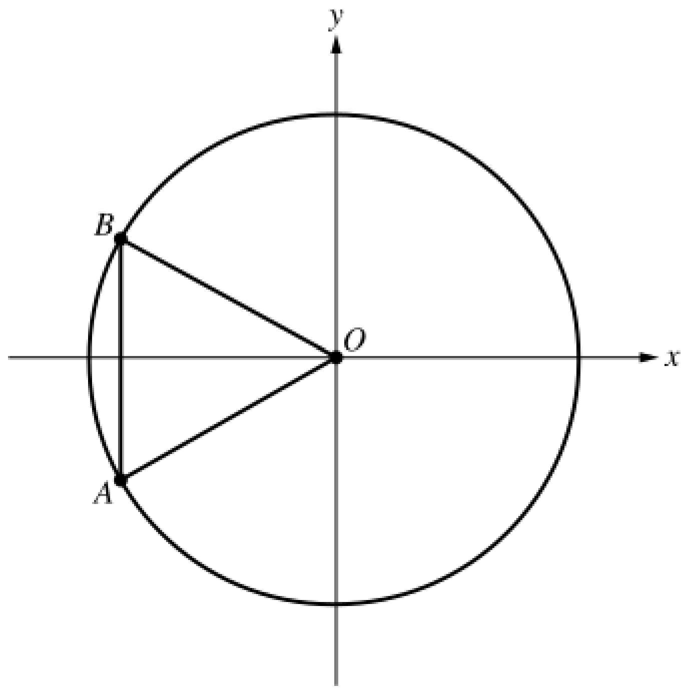
(D) f is increasing, and the graph of f is concave up.

Answer B

Correct. The function is decreasing on this interval. The rate of change is increasing on this interval. Therefore, the graph of f is concave up on this interval. Another way to think of this is as follows. In the xy-plane, let P be the point of intersection of the terminal ray of an angle θ in standard position and the unit circle. As θ increases on the interval $\pi < \theta < \frac{3\pi}{2}$, the y-coordinates of P are decreasing at an increasing rate. The behavior of the y-coordinates of P on the unit circle corresponds to the behavior of the sine function.

MCQ_Review_Unit_3

36.



The figure shows equilateral triangle ABO with sides of length 10 in the xy-plane. Segment AB is perpendicular to the x-axis. The terminal ray of an angle θ (not shown) in standard position passes through the point B. What is the value of $10 \sin \theta$?

(A) $-5\sqrt{3}$

(B) $-\frac{\sqrt{3}}{2}$

(C) $\frac{1}{2}$

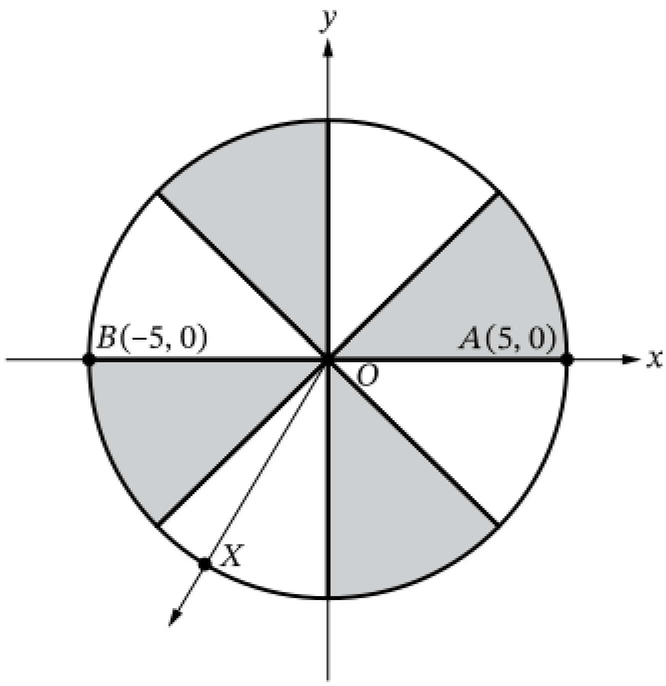
(D) 5

Answer D

Correct. The value of $10 \sin \theta$ is the y-coordinate of B. Because the triangle is equilateral, geometry of the triangle formed by O, B, and the point of intersection of segment AB with the x-axis can be used to find the coordinates of point B. If the point of intersection is called X, the length of segment OX is $5\sqrt{3}$, the length of segment BX is 5, and the radius OB has length 10. The y-coordinate of B is 5.

MCQ_Review_Unit_3

37.



The figure shows a spinner with eight congruent sectors in the xy-plane. The origin and three points are labeled. If the coordinates of X are $\left(-\frac{5}{2}, \frac{-5\sqrt{3}}{2}\right)$, what is the measure of angle XOB ?

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{3}$

(C) $\frac{7\pi}{6}$

(D) $\frac{4\pi}{3}$

Answer B

Correct. If the coordinates of X are $\left(-\frac{5}{2}, \frac{-5\sqrt{3}}{2}\right)$ and the radius of the circle is 5, then the coordinates can be written as $\left(5 \cdot -\frac{1}{2}, 5 \cdot -\frac{\sqrt{3}}{2}\right)$. Using known values for sine and cosine of multiples of $\frac{\pi}{6}$, an angle in standard position of $\frac{4\pi}{3}$ gives these values. Angle XOB can be found as $\frac{4\pi}{3} - \pi$.

38. The functions f and g are given by $f(x) = \cos x$ and $g(x) = \sin x$. In the xy-plane, for how much of the interval $0 \leq x \leq 8\pi$ are the graphs of f and g both concave up?

(A) 0 units of x
(B) $\frac{\pi}{2}$ units of x
(C) π units of x
(D) 2π units of x ✓

Answer D

Correct. Both f and g have periods of 2π. The graphs of both functions are concave up on the same interval for 25% or $\frac{\pi}{2}$ units of every period. The interval $0 \leq x \leq 8\pi$ contains 4 periods of the functions. $4\left(\frac{\pi}{2}\right) = 2\pi$. Therefore, both graphs are concave up on 2π units of x.

39. Which of the following describes the relationship between the graphs of $f(x) = \cos x$ and $g(x) = \sin x$ in the xy-plane?

(A) The graph of g is a horizontal translation of the graph of f. ✓
(B) The graph of g is a vertical translation of the graph of f.
(C) The graph of g is a horizontal dilation of the graph of f.
(D) The graph of g is a vertical dilation of the graph of f.

Answer A

Correct. It is known that $\cos\theta = \sin\left(\theta + \frac{\pi}{2}\right)$. The graph of g is a horizontal translation of the graph of f. It is also true that the graph of f is a horizontal translation of the graph of g.

40. The function f is given by $f(x) = \sin(x + c)$, where c is a constant. Which of the following statements is true?

(A) If $c = 0$, then f is an even function.
(B) If $c = \frac{\pi}{2}$, then f is an even function. ✓
(C) If $c = 2\pi$, then f is an even function.
(D) There is no value of c for which f is an even function.

Answer B

Correct. If $c = \frac{\pi}{2}$, then $f(x) = \sin\left(x + \frac{\pi}{2}\right)$. This function is an equivalent form for $f(x) = \cos x$, which is an even function.

41.

Time t (hours past 12 midnight)	**Temperature** ($^\circ$F)
0	23
2	22
4	23
6	26
8	30
10	34
12	37
14	38
16	37
18	34
20	30
22	26

The table gives temperatures, in degrees Fahrenheit, in a certain town on a given day. The sinusoidal function $F(t) = 8\cos\left(\frac{\pi}{12}(t+c)\right) + 30$ models the data, where $F(t)$ is the temperature, in $^\circ\mathrm{F}$, at time t, in hours past 12 midnight, and c is a constant. Which of the following is true about the value of c ?

(A) The value of c is -2 because this accounts for a phase shift of $g(t) = 8\cos\left(\frac{\pi}{12}t\right) + 30$ that aligns a minimum value of the data set with a minimum value of g.

(B) The value of c is -2 because this accounts for a vertical shift of $g(t) = 8\cos\left(\frac{\pi}{12}t\right) + 30$ that aligns a minimum value of the data set with a minimum value of g.

(C) The value of c is -14 because this accounts for a phase shift of $g(t) = 8\cos\left(\frac{\pi}{12}t\right) + 30$ that aligns a maximum value of the data set with a maximum value of g. ✓

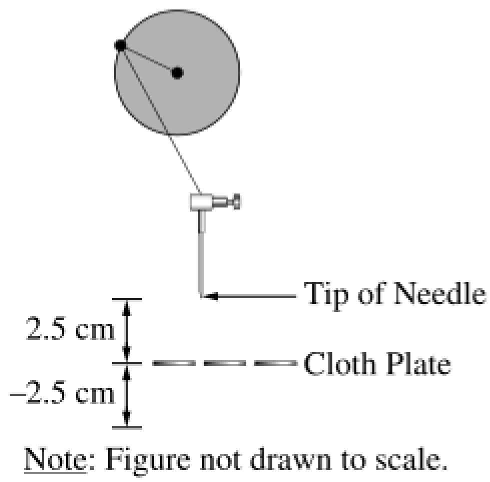
(D) The value of c is -14 because this accounts for a vertical shift of $g(t) = 8\cos\left(\frac{\pi}{12}t\right) + 30$ that aligns a maximum value of the data set with a maximum value of g.

MCQ_Review_Unit_3

Answer C

Correct. The input-output pair $(14,38)$ from the table gives a maximum temperature value. This aligns with a maximum value of the sinusoidal model $F(t) = 8\cos\left(\frac{\pi}{12}(t-14)\right) + 30$ due to a horizontal phase shift of 14 units to the right. The value of c is -14.

42.



Time t (seconds)	**Displacement of Tip from Cloth Plate (centimeters)**
0	2.5
$\frac{1}{24}$	-2.5

The vertical motion of the tip of a needle on a sewing machine is periodic with respect to time and can be modeled by a sinusoidal function. The figure shows the location of the tip of the needle in relation to a cloth plate. The tip of the needle moves straight up and down above and below the cloth plate. The table gives a consecutive maximum and minimum displacement of the tip at two times. The function d models the displacement, in centimeters, of the tip of the needle from the cloth plate at time t, in seconds. Which of the following expressions could define $d(t)$?

MCQ_Review_Unit_3

(A) $2.5\cos\left(\frac{1}{12}t\right)$

(B) $2.5\cos\left(\frac{\pi}{6}t\right)$

(C) $2.5\cos(24\pi t)$

(D) $2.5\cos(48\pi t)$

Answer C

Correct. The amplitude of the function is $\frac{2.5-(-2.5)}{2}=2.5$. The period of the function is $2\cdot\frac{1}{24}=\frac{1}{12}$, which represents the time, in seconds, between consecutive maxima. Therefore, the coefficient of t should satisfy $\frac{2\pi}{b}=\frac{1}{12}$, so $b=24\pi$.

43. The point P has polar coordinates $\left(10,\frac{5\pi}{6}\right)$. Which of the following is the location of point P in rectangular coordinates?

(A) $\left(-5\sqrt{3},5\right)$

(B) $\left(-5,5\sqrt{3}\right)$

(C) $\left(5\sqrt{3},5\right)$

(D) $\left(5\sqrt{3},-5\right)$

Answer A

Correct. The polar coordinates can be converted to rectangular coordinates using $(r\cos\theta, r\sin\theta)$. Therefore, $x=10\cos\left(\frac{5\pi}{6}\right)=10\left(-\frac{\sqrt{3}}{2}\right)=-5\sqrt{3}$ and $y=10\sin\left(\frac{5\pi}{6}\right)=10\left(\frac{1}{2}\right)=5$.

44. In the tidal area of a certain city, a sinusoidal function $f(x)=a\sin(b(x+c))+d$, where a, b, c, and d are constants, is used to model one cycle of high and low tides. The maximum value of the tide is 8.88 feet, and the minimum value of the tide is 0.54 feet in that cycle. If the values of b, c, and d have already been determined to fit the data, which of the following would best define $f(x)$?

(A) $4.17\sin(b(x+c))+d$

(B) $4.44\sin(b(x+c))+d$

(C) $4.71\sin(b(x+c))+d$

(D) $8.34\sin(b(x+c))+d$

MCQ_Review_Unit_3

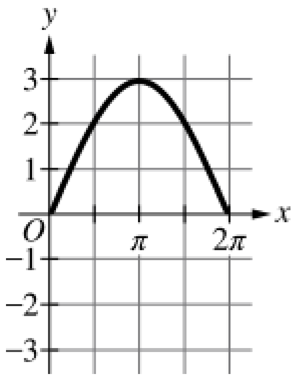
Answer A

Correct. The value of a relates to the amplitude (half of the difference between the maximum and minimum values) of the function: $\frac{8.88-0.54}{2}$

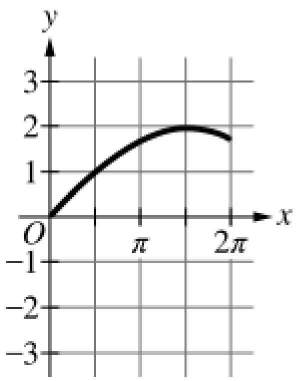
45. Which of the following is the graph of $f(x) = 3\sin(2x)$ in the xy-plane?

MCQ_Review_Unit_3

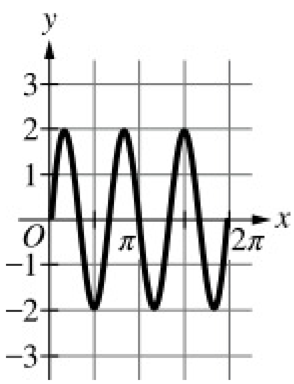
(A)



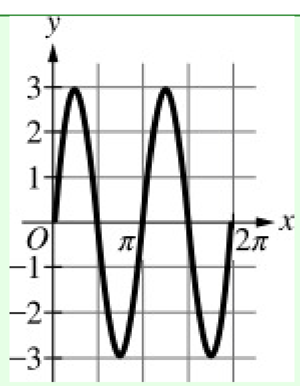
(B)



(C)



(D)



Answer D

Correct. The amplitude is 3, and the period is $\frac{2\pi}{2} = \pi$, making this the graph of function f.

46. The function f is given by $f(x) = \sin x$. In the xy-plane, the graph of the function g is the image of the graph of f after a translation of $\frac{\pi}{6}$ units to the left. Which of the following could define $g(x)$?

(A) $\sin x + \frac{\pi}{6}$

(B) $\sin\left(x + \frac{\pi}{6}\right)$

(C) $\sin x - \frac{\pi}{6}$

(D) $\sin\left(x - \frac{\pi}{6}\right)$

Answer B

Correct. The graph of the additive transformation $g(x) = \sin(x + c)$ of the sine function $f(x) = \sin x$ is a horizontal translation of the graph of f by $-c$ units. Because the translation is $-\frac{\pi}{6}$ units, $c = \frac{\pi}{6}$.

47.

Month	1	2	3	4	5	6	7	8	9
Temperature (degrees Celsius)	6.1	-5.5	-6.0	10.0	17.2	25.6	30.6	32.2	26.1

The table gives the maximum temperature, in degrees Celsius, on the first day of each of nine months in a certain city. The function f given by $f(\theta) = a\sin(b(\theta + c)) + d$, where a, b, c, and d are constants, is used to model these data with θ representing the number of the month. Assume that the period of f is 12 months. Based on the data in the table, which of the following is the best value for d ?

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(A) $\frac{\pi}{6}$

(B) 13

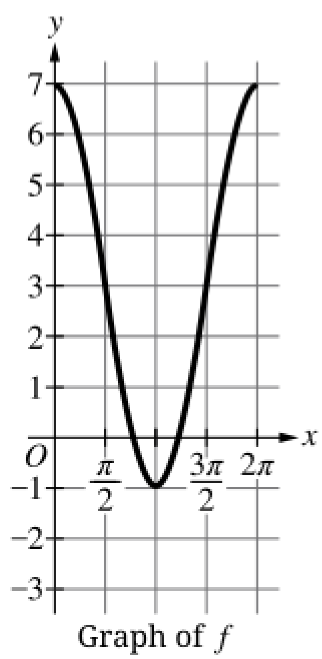
(C) 19

(D) 38

Answer B

Correct. The value of d is the average of the maximum and minimum values in the table.
$\frac{32.2+-5.5}{2} \approx 13$

48.



A portion of the graph of a sinusoidal function f in the xy-plane is given for $0 \leq x \leq 2\pi$. Which of the following could define $f(x)$?

(A) $3 + 4\cos x$

(B) $3 + 4\sin x$

(C) $4 + 3\cos x$

(D) $4 + 3\sin x$

Answer A

Correct. $\frac{7+(-1)}{2} = 3$, so the midline of the graph is at $y = 3$, which means a vertical shift up of 3. $\frac{7-(-1)}{2} = 4$, so the amplitude is 4. Because the graph has maxima at $x = 0$ and $x = 2\pi$ and the function options involve no phase shifts, a cosine function is the best option.

49. The function p is given by $p(\theta) = 3\tan\left(\frac{\pi}{2}(\theta+1)\right) - 4$. What is the period of p ?

(A) $\frac{2}{\pi}$

(B) $\frac{\pi}{2}$

(C) 2 ✓

(D) 4

Answer C

Correct. If b is the coefficient of θ, then $\left|\frac{1}{b}\right|\pi$ is the period of this transformation of $y = \tan\theta$. Therefore, the period is $\left|\frac{1}{\pi/2}\right|\pi = 2$.

50. The function g is given by $g(\theta) = \tan(2\pi\theta)$. Which of the following statements about the graph of g in the xy -plane is true?

(A) The vertical asymptotes of the graph of g occur at input values $\theta = 0 + \frac{1}{2}k$, where k is an integer.

(B) The vertical asymptotes of the graph of g occur at input values $\theta = \frac{1}{4} + \frac{1}{2}k$, where k is an integer. ✓

(C) The vertical asymptotes of the graph of g occur at input values $\theta = \frac{1}{4} + k$, where k is an integer.

(D) The vertical asymptotes of the graph of g occur at input values $\theta = \frac{1}{2} + 2k$, where k is an integer.

Answer B

Correct. The period of the function g is $\frac{\pi}{2\pi} = \frac{1}{2}$. The graph of g has vertical asymptotes where $\tan(2\pi\theta)$ is undefined. This occurs where $\cos(2\pi\theta) = 0$, which is at $\theta = \frac{1}{4}$, $\theta = \frac{3}{4}$, $\theta = \frac{5}{4}$, etc. The distance between consecutive vertical asymptotes is $\frac{1}{2}$.

MCQ_Review_Unit_3

51. The function f is given by $f(x) = a\tan(bx)$, where a and b are constants. Which of the following statements is true about the period of f ?

(A) Both the value of a and the value of b have an impact on the period of f.

(B) Only the value of a has an impact on the period of f.

(C) Only the value of b has an impact on the period of f.

(D) Neither the value of a nor the value of b has an impact on the period of f.

Answer C

Correct. The coefficient b of the input variable produces a multiplicative transformation resulting in a horizontal dilation that impacts the period of the function.

52. In the xy-plane, an angle θ, in standard position, has a measure of $\theta = \frac{\pi}{3}$. Which of the following is true?

(A) The slope of the terminal ray of the angle is $\frac{1}{2}$.

(B) The slope of the terminal ray of the angle is $\frac{1}{\sqrt{3}}$.

(C) The slope of the terminal ray of the angle is $\frac{\sqrt{3}}{2}$.

(D) The slope of the terminal ray of the angle is $\sqrt{3}$.

Answer D

Correct. The slope of the terminal ray is given by $\tan\left(\frac{\pi}{3}\right) = \frac{\sin(\pi/3)}{\cos(\pi/3)}$.

53. The function g is given by $g(x) = \tan x$. What are all solutions to $g(x) = 3$?

(A) $x = \arctan 3$ and $x = \pi + \arctan 3$ only

(B) $x = \arctan 3$ and $x = \pi + \arctan(3 + \pi)$ only

(C) $x = 2\pi k + \arctan 3$ only, where k is any integer

(D) $x = \pi k + \arctan 3$, where k is any integer

Answer D

Correct. Because the tangent function has a period of π, this represents the general solution to $g(x) = 3$.

MCQ_Review_Unit_3

54.

x	y
-1	$-\frac{\pi}{2}$
$-\frac{1}{2}$	$-\frac{\pi}{6}$
0	0
$\frac{1}{2}$	$\frac{\pi}{6}$
1	$\frac{\pi}{2}$

The table gives ordered pairs (x, y) that are solutions to which of the following?

(A) $y = \cos x$

(B) $y = \cos^{-1} x$

(C) $y = \sin x$

(D) $y = \sin^{-1} x$

Answer D

Correct. The inverse sine function is appropriate for the input values and output values of the table.

55. The number of moose in a park is modeled by the sinusoidal function M given by $M(t) = 174 + 150\sin\left(\frac{2\pi}{365}t\right)$, where t is the number of days since January 1. Which of the following statements is true?

(A) M is decreasing on the interval $0 < t < 182$.

(B) M is decreasing on the interval $92 < t < 273$.

(C) M is decreasing on the interval $183 < t < 365$.

(D) M is decreasing on the intervals $0 < t < 91$ and $274 < t < 365$.

MCQ_Review_Unit_3

Answer B

Correct. The period of the sinusoidal function M is 365 days. The function has maximum and minimum values, respectively, one quarter and three quarters of the way through the period. Between these two values, the function is decreasing.

56. What are all values of θ, for $0 \leq \theta < 2\pi$, where $2\sin^2\theta = -\sin\theta$?

(A) $0, \pi, \frac{\pi}{6}$, and $\frac{5\pi}{6}$

(B) $0, \pi, \frac{7\pi}{6}$, and $\frac{11\pi}{6}$

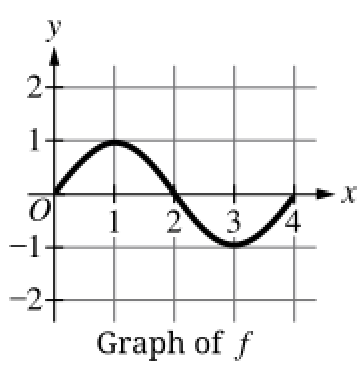
(C) $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}$, and $\frac{5\pi}{3}$

(D) $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{2\pi}{3}$, and $\frac{4\pi}{3}$

Answer B

Correct. The trigonometric equation is equivalent to $2\sin^2\theta + \sin\theta = 0$ and factors to $\sin\theta(2\sin\theta + 1) = 0$. The solutions are the solutions of $\sin\theta = 0$ and $2\sin\theta + 1 = 0 \rightarrow \sin\theta = -\frac{1}{2}$ on the interval $0 \leq \theta < 2\pi$.

57.



The graph of the sinusoidal function f is given in the xy-plane. What is the length of the largest interval of input values of f on which an inverse function of f can be constructed?

MCQ_Review_Unit_3

(A) 1

(B) 2

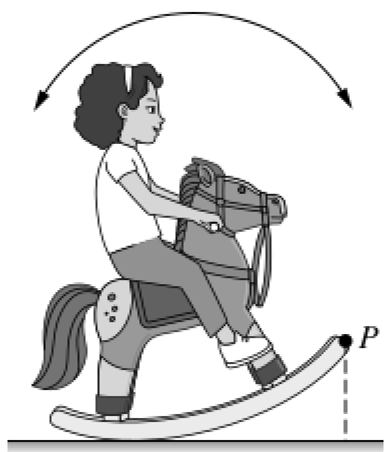
(C) 4

(D) There is no interval larger than a single point on which f is invertible.

Answer B

Correct. The largest interval of the sinusoidal function f on which no y-values are repeated and all possible y-values are represented has length 2. This is the interval from $x = 1$ to $x = 3$.

58.



Note: Figure not drawn to scale.

The figure shows a child leaning backward and forward when riding a rocking horse. The height, in inches, between the level ground and point P is given by $h(t) = 6 + 6\cos\left(\frac{\pi}{3}t\right)$, where t is the time since the child first leans farthest back, in seconds. If the child rides the rocking horse for 5 minutes (300 seconds), how many times will point P touch the ground?

(A) 1

(B) 50

(C) 100

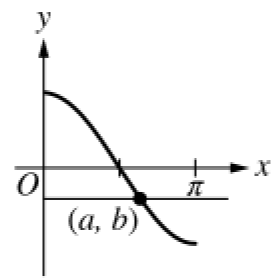
(D) 286

Answer B

Correct. The period of function h is $\frac{2\pi}{\pi/3} = 6$ seconds. The solution can be found by dividing 300

seconds (the equivalent of 5 minutes) by the period.

59.



The graphs of a trigonometric function f and a line are given in the xy-plane. The point of intersection (a, b) occurs where $f(x) = b$. For which of the following systems of equations could the point (b, a) be a solution?

(A) $\begin{cases} y = \cos^{-1} x \\ y = b \end{cases}$

(B) $\begin{cases} y = \cos^{-1} x \\ x = b \end{cases}$

(C) $\begin{cases} y = \sin^{-1} x \\ y = b \end{cases}$

(D) $\begin{cases} y = \sin^{-1} x \\ x = b \end{cases}$

Answer B

Correct. By examining the figure provided, f could be $y = \cos x$, and the line is $y = b$. Using inverse properties, if $x = b$, then $\cos^{-1} b = a$. Therefore, (b, a) is the solution of this system and the point of intersection of the graphs of $x = b$ and $y = \cos^{-1} x$.

60. The function f is given by $f(x) = 5 + 7\cos x$. For what value of x on the interval $\pi < x < 2\pi$ does $f(x) = 0$?

MCQ_Review_Unit_3

(A) $\arccos\left(-\frac{5}{7}\right)$

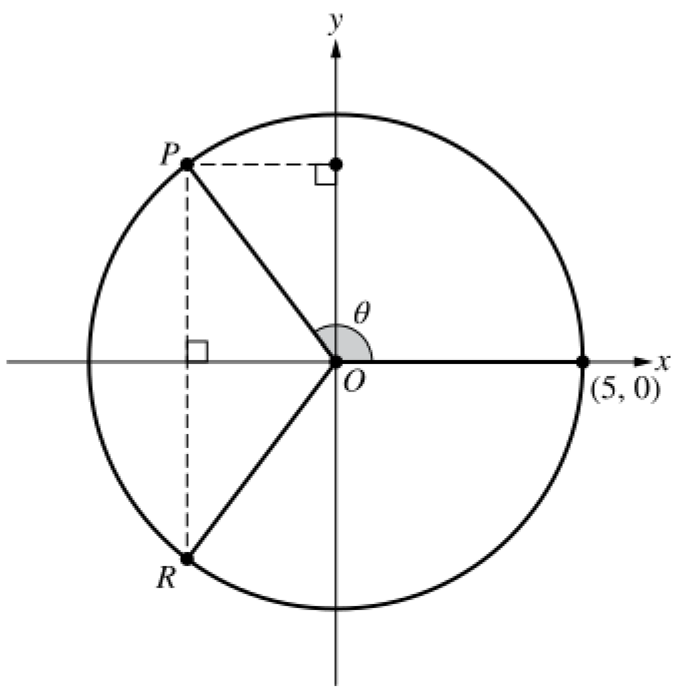
(B) $\pi + \arccos\left(\frac{5}{7}\right)$

(C) $\pi + \arccos\left(-\frac{5}{7}\right)$

(D) $2\pi - \arccos\left(\frac{5}{7}\right)$

Answer B

Correct. This is the result of finding the Quadrant I reference angle of $\frac{5}{7}$, then adjusting to Quadrant III by adding π to account for the domain restriction of $\pi < x < 2\pi$.



The figure shows a circle centered at the origin with an angle of measure θ radians in standard position. The terminal ray of the angle intersects the circle at point P, and point R also lies on the circle. The coordinates of P are (x, y), and the coordinates of R are $(x, -y)$.

61. Which of the following is true about the sine of θ ?

(A) $\sin\theta = \frac{x}{5}$, because it is the ratio of the horizontal displacement of P from the y-axis to the distance between the origin and P.

(B) $\sin\theta = \frac{x}{5}$, because it is the ratio of the horizontal displacement of R from the y-axis to the distance between the origin and R.

(C) $\sin\theta = \frac{-y}{5}$, because it is the ratio of the vertical displacement of R from the x-axis to the distance between the origin and R.

(D) $\sin\theta = \frac{y}{5}$, because it is the ratio of the vertical displacement of P from the x-axis to the distance between the origin and P.

Answer D

Correct. The ratio of the vertical displacement of P from the x-axis to the distance between the origin and P is $\sin\theta$.

---

62. Let $f(x) = 1 + 3\sec x$ and $g(x) = -5$. In the xy-plane, what are the x-coordinates of the points of intersection of the graphs of f and g for $0 \le x < 2\pi$?

(A) $x = \frac{\pi}{3}$ and $x = \frac{5\pi}{3}$

(B) $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$

(C) $x = \frac{2\pi}{3}$ and $x = \frac{4\pi}{3}$

(D) $x = \frac{7\pi}{6}$ and $x = \frac{11\pi}{6}$

Answer C

Correct. Finding the x-coordinates of the points of intersection of the graphs of f and g is equivalent to solving the equation $f(x) = g(x)$ for x. Simplifying the equation results in $\sec x = -2$. This equation is equivalent to $\cos x = -\frac{1}{2}$. The solutions on the interval $0 \le x < 2\pi$ are $x = \frac{2\pi}{3}$ and $x = \frac{4\pi}{3}$.

63. The function f is defined by $f(\theta) = \cos(2\theta)$. Which of the following is an equivalent expression for $f(\theta)$?

(A) 1

(B) $1 - 2\cos^2\theta$

(C) $1 - 2\sin^2\theta$

(D) $2\cos\theta\sin\theta$

MCQ_Review_Unit_3

Answer C

Correct. Using the sum identity for cosine, $f(\theta) = \cos(2\theta) = \cos(\theta + \theta)$, which equals $\cos\theta\cos\theta - \sin\theta\sin\theta = \cos^2\theta - \sin^2\theta$. Using the Pythagorean identity, this can be rewritten as $\cos^2\theta - \sin^2\theta = 1 - \sin^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$.

64. The function f is given by $f(x) = \cos\left(x + \frac{\pi}{6}\right)$. The solutions to which of the following equations on the interval $0 \le x \le 2\pi$ are the solutions to $f(x) = \frac{1}{2}$ on the interval $0 \le x \le 2\pi$?

(A) $\sqrt{3}\cos x - \sin x = 1$ ✓

(B) $\sqrt{3}\cos x + \sin x = 1$

(C) $\sqrt{3}\sin x - \cos x = 1$

(D) $\sqrt{3}\sin x + \cos x = 1$

Answer A

Correct. By the sum identity for cosine, $f(x) = \cos\left(x + \frac{\pi}{6}\right) = \cos x\cos\left(\frac{\pi}{6}\right) - \sin x\sin\left(\frac{\pi}{6}\right)$, which is equivalent to $\cos x\left(\frac{\sqrt{3}}{2}\right) - \sin x\left(\frac{1}{2}\right)$. Setting this expression equal to $\frac{1}{2}$ gives $\cos x\left(\frac{\sqrt{3}}{2}\right) - \sin x\left(\frac{1}{2}\right) = \frac{1}{2} \rightarrow \cos x\left(\sqrt{3}\right) - \sin x = 1$. Therefore, on the interval $0 \le x \le 2\pi$, the solutions to this equation are also the solutions to $f(x) = \frac{1}{2}$.

65. In the xy-plane, the terminal ray of an angle in standard position intersects the unit circle at the point with coordinates (a, b). The terminal ray of a second angle in standard position intersects the circle at the point with coordinates (c, d). If the measure of the second angle is twice the measure of the first angle, what are the coordinates c and d, in terms of a and b ?

(A) $c = -b$ and $d = -a$

(B) $c = 2a$ and $d = 2b$

(C) $c = a^2 + b^2$ and $d = 2ab$

(D) $c = a^2 - b^2$ and $d = 2ab$ ✓

MCQ_Review_Unit_3

Answer D

Correct. The first angle is θ, and the second angle is 2θ. Because this is the unit circle, the coordinates are $a = \cos\theta$, $b = \sin\theta$, $c = \cos(2\theta)$, and $d = \sin(2\theta)$. Using the double-angle identities, $c = \cos(2\theta) = \cos^2\theta - \sin^2\theta = a^2 - b^2$ and $d = \sin(2\theta) = 2\sin\theta\cos\theta = 2ab$.

66. The function g is defined by $g(x) = \sec^2 x + \tan x$. What are all solutions to $g(x) = 1$ on the interval $0 \le x \le 2\pi$?

(A) $x = 0$, $x = \frac{3\pi}{4}$, $x = \pi$, $x = \frac{7\pi}{4}$, and $x = 2\pi$ only ✓

(B) $x = \frac{\pi}{4}$, $x = \frac{\pi}{2}$, $x = \frac{5\pi}{4}$, and $x = \frac{3\pi}{2}$ only

(C) $x = \pi k$ and $x = -\frac{\pi}{4} + \pi k$, where k is any integer

(D) $x = \frac{\pi}{2} + \pi k$ and $x = \frac{\pi}{4} + \pi k$, where k is any integer

Answer A

Correct. Using the Pythagorean identity $\sec^2 x = 1 + \tan^2 x$, the expression for $g(x)$ can be rewritten in an equivalent form to solve the equation.$1 + \tan^2 x + \tan x = 1 \rightarrow \tan x(\tan x + 1) = 0$, which means $\tan x = 0$ or $\tan x + 1 = 0$. Solving both of these equations on the interval $0 \le x \le 2\pi$ provides a complete solution. $\tan x = 0$ when $x = 0$, $x = \pi$, or $x = 2\pi$; $\tan x + 1 = 0$ means $\tan x = -1$ when $x = \frac{3\pi}{4}$ or $x = \frac{7\pi}{4}$.

67. The function f is given by $f(x) = \sin^2 x + \cos x + 1$. The solutions to which of the following equations are also the solutions to $f(x) = 0$?

(A) $\cos^2 x + \cos x = 0$

(B) $\cos^2 x - \cos x - 2 = 0$ ✓

(C) $\cos^2 x + \cos x + 2 = 0$

(D) $\sin^2 x - \sin x + 2 = 0$

Answer B

Correct. By the Pythagorean identity, $\sin^2 x = 1 - \cos^2 x$. Therefore, $f(x) = 1 - \cos^2 x + \cos x + 1 = -\cos^2 x + \cos x + 2$. Setting $f(x) = 0$, this can be written in an equivalent form as $\cos^2 x - \cos x - 2 = 0$.

68. The function f is given by $f(x) = \cos x\left(1 + \tan^2 x\right)$. Which of the following expressions is equivalent to $f(x)$?

(A) $\sec x$

(B) $\cos^3 x$

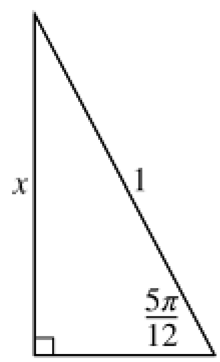
(C) $\tan x \sec x$

(D) $\cot x \csc x$

Answer A

Correct. Using the Pythagorean identity, $f(x) = \cos x\left(1 + \tan^2 x\right) = (\cos x)\left(\sec^2 x\right)$. Since cosine and secant are reciprocal functions, this is equivalent to $(\cos x)(\sec x)(\sec x) = 1 \cdot \sec x$.

69.



The figure is a right triangle with a hypotenuse of length 1, a side of length x, and the angle opposite that side with measure $\frac{5\pi}{12}$. Using the fact that $\frac{5\pi}{12} = \frac{3\pi}{12} + \frac{2\pi}{12}$, which of the following is the value of x ?

MCQ_Review_Unit_3

(A) $\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

(B) $\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

(C) $\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)-\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

(D) $\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)-\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

Answer B

Correct. Using the right triangle definition for sine, $\sin\left(\frac{5\pi}{12}\right)=\frac{x}{1}$. Using the sum identity for sine, $x=\sin\left(\frac{5\pi}{12}\right)=\sin\left(\frac{3\pi}{12}+\frac{2\pi}{12}\right)=\sin\left(\frac{3\pi}{12}\right)\cos\left(\frac{2\pi}{12}\right)+\cos\left(\frac{3\pi}{12}\right)\sin\left(\frac{2\pi}{12}\right)$. This is equal to $\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$, where $\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$.

70. The function f is given by $f(x)=\sin\left(x+\frac{4\pi}{3}\right)$. What is the value of $f\left(\frac{3\pi}{4}\right)$?

(A) $\frac{-\sqrt{6}-\sqrt{2}}{4}$

(B) $\frac{\sqrt{2}+\sqrt{6}}{4}$

(C) $\frac{\sqrt{2}-\sqrt{6}}{4}$

(D) $\frac{\sqrt{6}-\sqrt{2}}{4}$

Answer D

Correct. By applying the sum identity for sine, $f(x)=\sin\left(x+\frac{4\pi}{3}\right)=\sin x\cos\left(\frac{4\pi}{3}\right)+\cos x\sin\left(\frac{4\pi}{3}\right)$. Evaluating this expression for $x=\frac{3\pi}{4}$ results in $\sin\left(\frac{3\pi}{4}\right)\cos\left(\frac{4\pi}{3}\right)+\cos\left(\frac{3\pi}{4}\right)\sin\left(\frac{4\pi}{3}\right)$, which gives $\frac{\sqrt{2}}{2}\cdot-\frac{1}{2}+-\frac{\sqrt{2}}{2}\cdot-\frac{\sqrt{3}}{2}=\frac{-\sqrt{2}+\sqrt{6}}{4}$. This can be rewritten as $\frac{\sqrt{6}-\sqrt{2}}{4}$.

71. The cosine function is a horizontal translation of the sine function with $\sin\left(x+\frac{\pi}{2}\right)=\cos x$. Which of the following identities can be used to verify this identity by direct substitution and evaluation without additional algebraic manipulation?

MCQ_Review_Unit_3

(A) The sum identity for sine ✓

(B) The sum identity for cosine

(C) The double-angle identity for sine

(D) The double-angle identity for cosine

Answer A

Correct. By direct substitution, $\sin\left(x+\frac{\pi}{2}\right)=\sin x\cos\left(\frac{\pi}{2}\right)+\cos x\sin\left(\frac{\pi}{2}\right)$, and by evaluation this is $\sin x\cdot 0+\cos x\cdot 1=\cos x$.

72. The function g is given by $g(x)=7\sin(2x)$. Which of the following is an equivalent form for $g(x)$?

(A) $g(x)=14\cos x\sin x$ ✓

(B) $g(x)=(7\cos x)(7\sin x)$

(C) $g(x)=7\cos^2 x-7\sin^2 x$

(D) $g(x)=7-14\sin^2 x$

Answer A

Correct. Function g can be written in an equivalent form by applying the sum (double-angle) identity for sine, $\sin(2x)=\sin(x+x)=\sin x\cos x+\cos x\sin x=2\sin x\cos x$, then multiplying by the coefficient 7.

73. What are all values of θ, $-\pi\leq\theta\leq\pi$, for which $2\cos\theta>-1$ and $2\sin\theta>\sqrt{3}$?

(A) $-\frac{5\pi}{6}<\theta<\frac{5\pi}{6}$

(B) $\frac{\pi}{6}<\theta<\frac{5\pi}{6}$ only

(C) $-\frac{2\pi}{3}<\theta<\frac{2\pi}{3}$ only

(D) $\frac{\pi}{3}<\theta<\frac{2\pi}{3}$ only ✓

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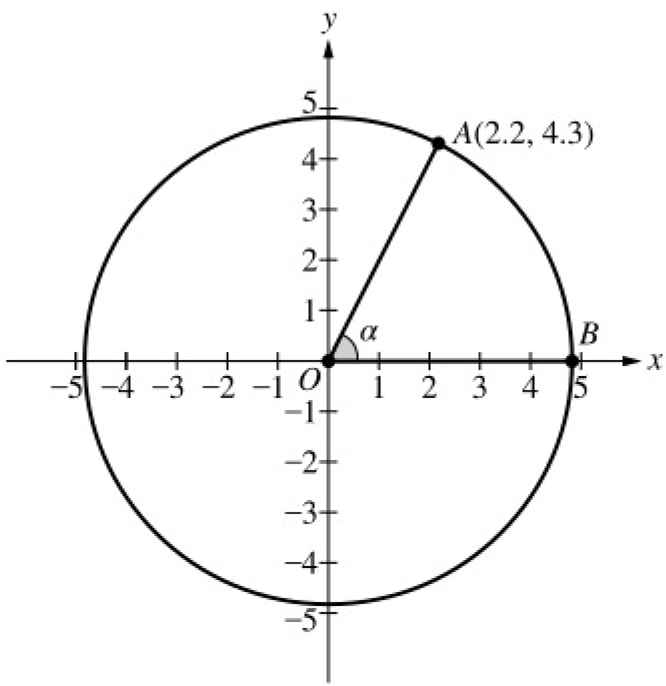
Answer D

Correct. Within the interval of $-\pi \leq \theta \leq \pi$, $2\cos\theta > -1$ for all values of θ on the interval $-\frac{2\pi}{3} < \theta < \frac{2\pi}{3}$ and $2\sin\theta > \sqrt{3}$ for all of θ on the interval $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$. The values of θ that are shared by both trigonometric inequalities on this interval are $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$.

74.



The figure gives a circle of radius 4.8 in the xy-plane with center at the origin, an angle α in standard position, and three labeled points. Which of the following is the value of $\sin\alpha$?

(A) $\frac{2.2}{4.8}$

(B) $\frac{4.3}{4.8}$

(C) 2.2

(D) 4.3

Answer B

Correct. The sine of an angle is the ratio of the vertical displacement of the point from the x-axis to the distance between the origin and the point.

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75. In the xy-plane, angle BAC is an angle in standard position with terminal ray AC, which intersects the unit circle at the point with coordinates $(0.4, -0.9)$. Which of the following descriptions is correct?

(A) The tangent of angle BAC is $-\frac{4}{9}$, and the slope of ray AC is $-\frac{4}{9}$.

(B) The tangent of angle BAC is $-\frac{4}{9}$, and the slope of ray AC is $-\frac{9}{4}$.

(C) The tangent of angle BAC is $-\frac{9}{4}$, and the slope of ray AC is $-\frac{9}{4}$.

(D) The tangent of angle BAC is $-\frac{9}{4}$, and the slope of ray AC is $\frac{4}{9}$.

Answer C

Correct. The tangent is the ratio of the vertical displacement of the point of intersection from the x-axis to the horizontal displacement of the point of intersection from the y-axis, which is also the slope of the terminal ray.

76. An angle θ is in standard position in the xy-plane. Which of the following is true about θ on the interval $0 \leq \theta \leq 2\pi$ if $\tan\theta = 1$?

(A) There is a value of θ on $0 \leq \theta \leq 2\pi$ for which $\tan\theta = 1$ in Quadrant I only.

(B) There are values of θ on $0 \leq \theta \leq 2\pi$ for which $\tan\theta = 1$ in Quadrants I and III only.

(C) There are values of θ on $0 \leq \theta \leq 2\pi$ for which $\tan\theta = 1$ in all four Quadrants.

(D) There is no value of θ on $0 \leq \theta \leq 2\pi$ for which $\tan\theta = 1$.

Answer B

Correct. The trigonometric function $\tan\theta$ is equal to 1 when $\frac{\sin\theta}{\cos\theta} = 1$ or $\sin\theta = \cos\theta$. This occurs at $\theta = \frac{\pi}{4}$ (Quadrant I) and $\theta = \frac{5\pi}{4}$ (Quadrant III).

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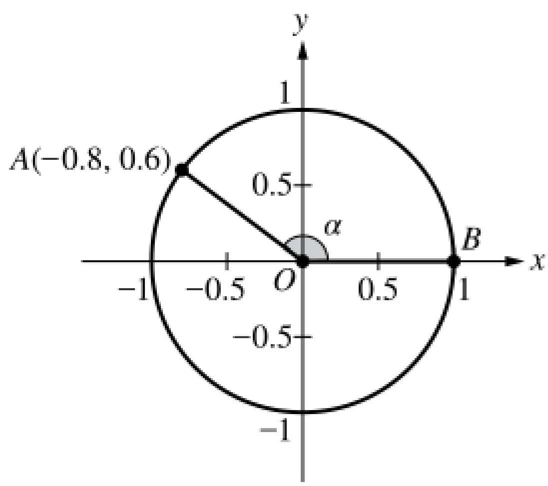
77.



The figure shows a unit circle in the xy-plane, an angle α in standard position, and three labeled points. Which of the following is the value of $\cos\alpha$?

(A) -0.8

(B) -0.6

(C) 0.6

(D) 0.8

Answer A

Correct. The cosine of an angle in standard position is the x-coordinate of the point on the terminal ray that intersects the unit circle, a circle of radius 1.

78. Consider the graph of the polar function $r = f(\theta)$, where $f(\theta) = 1 + 2\sin\theta$, in the polar coordinate system for $0 \le \theta \le 2\pi$. Which of the following statements is true about the distance between the point with polar coordinates $(f(\theta), \theta)$ and the origin?

(A) The distance is increasing for $0 \le \theta \le \frac{\pi}{2}$, because $f(\theta)$ is positive and increasing on the interval.

(B) The distance is increasing for $\frac{3\pi}{2} \le \theta \le \frac{11\pi}{6}$, because $f(\theta)$ is negative and increasing on the interval.

(C) The distance is decreasing for $0 \le \theta \le \frac{\pi}{2}$, because $f(\theta)$ is positive and decreasing on the interval.

(D) The distance is decreasing for $\frac{3\pi}{2} \le \theta \le \frac{11\pi}{6}$, because $f(\theta)$ is negative and decreasing on the interval.

MCQ_Review_Unit_3

Answer A

Correct. Because the polar function is positive and increasing on this interval, the distance between the point $(f(\theta), \theta)$ and the origin is increasing on this interval.

79. A polar function is given by $r = f(\theta) = -1 + \sin\theta$. As θ increases on the interval $0 < \theta < \frac{\pi}{2}$, which of the following is true about the points on the graph of $r = f(\theta)$ in the xy-plane?

(A) The points on the graph are above the x-axis and are getting closer to the origin.

(B) The points on the graph are above the x-axis and are getting farther from the origin.

(C) The points on the graph are below the x-axis and are getting closer to the origin. ✓

(D) The points on the graph are below the x-axis and are getting farther from the origin.

Answer C

Correct. On the interval $0 < \theta < \frac{\pi}{2}$, the polar function $r = f(\theta)$ is negative and increasing, which means that the distance between the point $(f(\theta), \theta)$ on the graph of the function and the origin is decreasing.

80. Consider the graph of the polar function $r = f(\theta)$, where $f(\theta) = \theta(\theta - 2)(\theta - 4)$, in the polar coordinate system for $0 \le \theta \le 4$. Which of the following statements is true?

(A) On the interval $2 < \theta < 2.1$, the distance between $(f(\theta), \theta)$ and the origin is increasing because the values of $f(\theta)$ are negative and decreasing. ✓

(B) On the interval $2 < \theta < 2.1$, the distance between $(f(\theta), \theta)$ and the origin is decreasing because the values of $f(\theta)$ are negative and decreasing.

(C) On the interval $2 < \theta < 2.1$, the distance between $(f(\theta), \theta)$ and the origin is increasing because the values of $f(\theta)$ are negative and increasing.

(D) On the interval $2 < \theta < 2.1$, the distance between $(f(\theta), \theta)$ and the origin is decreasing because the values of $f(\theta)$ are negative and increasing.

Answer A

Correct. Because the polar function $r = f(\theta)$ is negative and decreasing on the interval $2 < \theta < 2.1$, the distance between $(f(\theta), \theta)$ and the origin is increasing.

MCQ_Review_Unit_3

81. Consider the graph of the polar function $r = f(\theta)$, where $f(\theta) = 2\sin\theta - 1$, in the polar coordinate system. Which of the following descriptions is true?

(A) As θ increases from 0 to $\frac{\pi}{6}$, the polar function $r = f(\theta)$ is increasing, and the distance between the point $(f(\theta), \theta)$ on the curve and the origin is increasing.

(B) As θ increases from 0 to $\frac{\pi}{6}$, the polar function $r = f(\theta)$ is increasing, and the distance between the point $(f(\theta), \theta)$ on the curve and the origin is decreasing. ✓

(C) As θ increases from 0 to $\frac{\pi}{6}$, the polar function $r = f(\theta)$ is decreasing, and the distance between the point $(f(\theta), \theta)$ on the curve and the origin is increasing.

(D) As θ increases from 0 to $\frac{\pi}{6}$, the polar function $r = f(\theta)$ is decreasing, and the distance between the point $(f(\theta), \theta)$ on the curve and the origin is decreasing.

Answer B

Correct. On this interval, the polar function is negative and increasing. The value increases from -1 to 0, and the distance between a point on the curve and the origin is decreasing from 1 to 0.

82. The function f is given by $f(x) = 3\cos(\pi x)$, where $0 \le x \le 1$. Which of the following gives $f^{-1}(x)$?

(A) $3\cos^{-1}(\pi x)$, $-3 \le x \le 3$

(B) $\frac{1}{3\cos(\pi x)}$, $-3 \le x \le 3$

(C) $\pi\cos^{-1}\left(\frac{x}{3}\right)$, $-3 \le x \le 3$

(D) $\frac{1}{\pi}\cos^{-1}\left(\frac{x}{3}\right)$, $-3 \le x \le 3$ ✓

Answer D

Correct. The inverse function is found by switching the input and output values and applying inverse operations (including using inverse cosine) to solve for y. Note that the domain of this function is $-3 \le x \le 3$ and the range is $0 \le y \le 1$.

MCQ_Review_Unit_3

83. Part of a video game design involves the use of one period of a sinusoidal function as the path that a spaceship will follow across a rectangular video screen. The video screen has a width of 1000 pixels and a height of 600 pixels. The values $x = 0$ and $x = 1000$ represent the left and right sides of the screen, respectively. The values $y = 0$ and $y = 600$ represent the bottom and top sides of the screen, respectively.

The path of the spaceship begins on the left side of the screen, $x = 0$, and completes one period of a sinusoidal function by ending on the right side of the screen, $x = 1000$. During its path, the spaceship reaches its minimum height of $y = 200$ before reaching its maximum height of $y = 500$. If $y = f(x)$ models the path of the spaceship, which of the following could define $f(x)$?

(A) $-300\sin\left(\frac{\pi}{500}x\right) + 350$

(B) $-150\sin\left(\frac{\pi}{500}x\right) + 350$ ✓

(C) $150\sin\left(\frac{\pi}{500}x\right) + 350$

(D) $300\sin\left(\frac{\pi}{500}x\right) + 350$

Answer B

Correct. The amplitude is $\frac{500-200}{2}$. The vertical shift is $\frac{200+500}{2}$. The period is 1000, so $\frac{2\pi}{b} = 1000$ and b, the coefficient of x, equals $\frac{2\pi}{1000}$. Because the minimum height is reached before the maximum height, a reflection of the parent function $\sin x$ over the x-axis is needed, which requires the negative sign.

84. Consider the graph of the polar function $r = f(\theta)$, where $f(\theta) = \frac{3\theta^3+50}{2\theta^4+5}$, in the polar coordinate system. Which of the following is true?

(A) Because $\lim\limits_{\theta\to\infty} f(\theta) = -\infty$, points on the graph of $r = f(\theta)$ will be arbitrarily close to the origin for sufficiently large values of θ.

(B) Because $\lim\limits_{\theta\to\infty} f(\theta) = 0$, points on the graph of $r = f(\theta)$ will be arbitrarily close to the origin for sufficiently large values of θ. ✓

(C) Because $\lim\limits_{\theta\to\infty} f(\theta) = \frac{3}{2}$, points on the graph of $r = f(\theta)$ will be arbitrarily close to the polar curve $r = \frac{3}{2}$ for sufficiently large values of θ.

(D) Because $\lim\limits_{\theta\to\infty} f(\theta) = \infty$, points on the graph of $r = f(\theta)$ will be increasingly distant from the origin for sufficiently large values of θ.

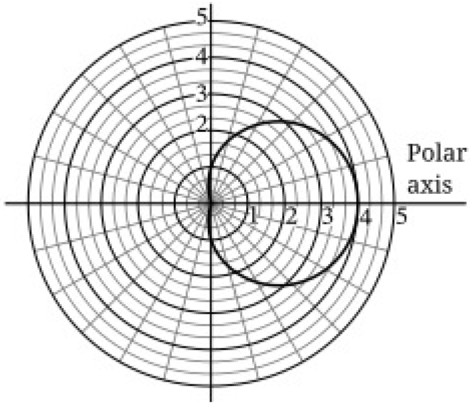
MCQ_Review_Unit_3

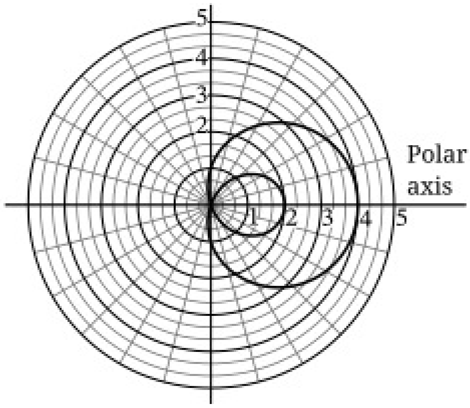
Answer B

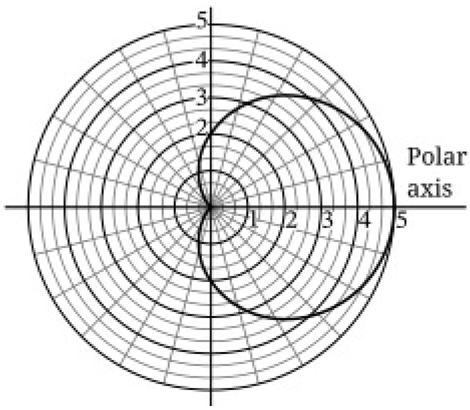
Correct. For sufficiently large values of θ, the polar function values decrease toward 0 and the points on the graph get arbitrarily close to the origin.

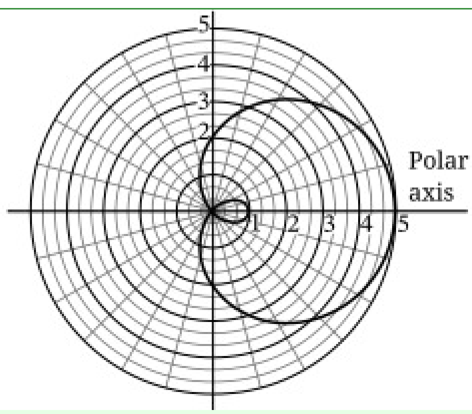
85. Which of the following is the graph of the polar function $r = f(\theta)$, where $f(\theta) = 3\cos\theta + 2$, in the polar coordinate system for $0 \leq \theta \leq 2\pi$?

MCQ_Review_Unit_3

(A) 

(B) 

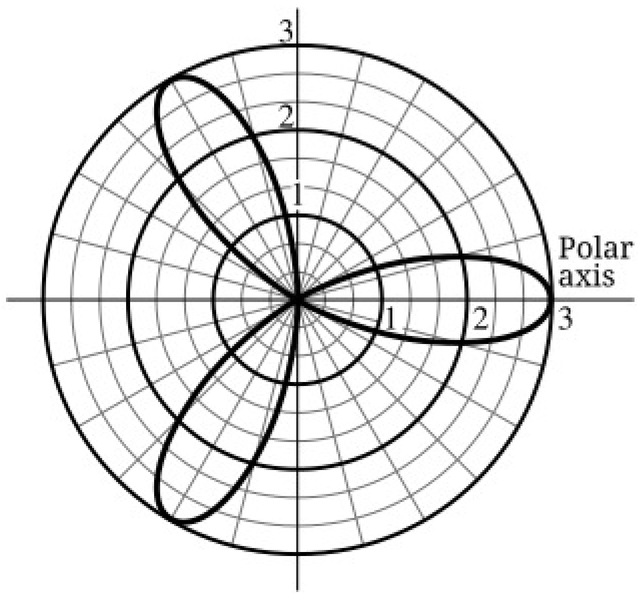
(C) 

(D) 

Answer D

Correct. Each point on the graph of the polar function $r = f(\theta)$ has polar coordinates $((f(\theta), \theta)$. Consider input values of $f(0)$, $f(\pi)$, and $f(2\pi)$, their corresponding output values, the symmetry of the graph over the polar axis, and the behavior of the polar function on the intervals $0 \leq \theta \leq \pi$ and $\pi \leq \theta \leq 2\pi$ to determine the shape of the curve.

86.



The graph of the polar function $r = f(\theta)$ is given in the polar coordinate system. Which of the following defines $f(\theta)$ for $0 \leq \theta \leq 2\pi$?

(A) $3 + \cos(3\theta)$

(B) $3\cos(3\theta)$

(C) $3 + \sin(3\theta)$

(D) $3\sin(3\theta)$

Answer B

Correct. By selecting and evaluating key values on the graph at $\theta = 0$, $\theta = \frac{2\pi}{3}$, and $\theta = \frac{4\pi}{3}$, and thinking about increasing and decreasing intervals of θ, the expression for $f(\theta)$ can be determined.

MCQ_Review_Unit_3

87. The polar function $r = f(\theta)$, where $f(\theta) = \frac{2}{\theta}$, is defined for $\theta \geq 1$. Which of the following describes the graph of $r = f(\theta)$ in the polar coordinate system?

(A) The graph of $r = f(\theta)$ is a line along the polar axis (x-axis) for which increasing angle measures correspond to decreasing radii.

(B) The graph of $r = f(\theta)$ is a line along the polar axis (x-axis) for which increasing angle measures correspond to increasing radii.

(C) The graph of $r = f(\theta)$ is a spiral for which increasing angle measures correspond to decreasing radii.

(D) The graph of $r = f(\theta)$ is a spiral for which increasing angle measures correspond to increasing radii.

Answer C

Correct. The spiral starts at the point with polar coordinates $(r, \theta) = (2,1)$. As the spiral traces through increasing angle measures, the values of the radii decrease. The value of $\frac{2}{\theta}$ decreases (gets closer to 0) as the value of θ increases. The spiral gets closer to the origin.

88. Let $r = f(\theta)$ be a polar function, where $f(\theta) = 5$. The graph of $r = f(\theta)$ for $0 \leq \theta \leq \pi$ appears as a semicircle in the polar coordinate system. The graph of which of the following polar functions in the polar coordinate system is the same semicircle?

(A) $r = g(\theta)$, where $g(\theta) = -5$ for $0 \leq \theta \leq \pi$

(B) $r = h(\theta)$, where $h(\theta) = \frac{1}{5}$ for $0 \leq \theta \leq \pi$

(C) $r = k(\theta)$, where $k(\theta) = -5$ for $\pi \leq \theta \leq 2\pi$

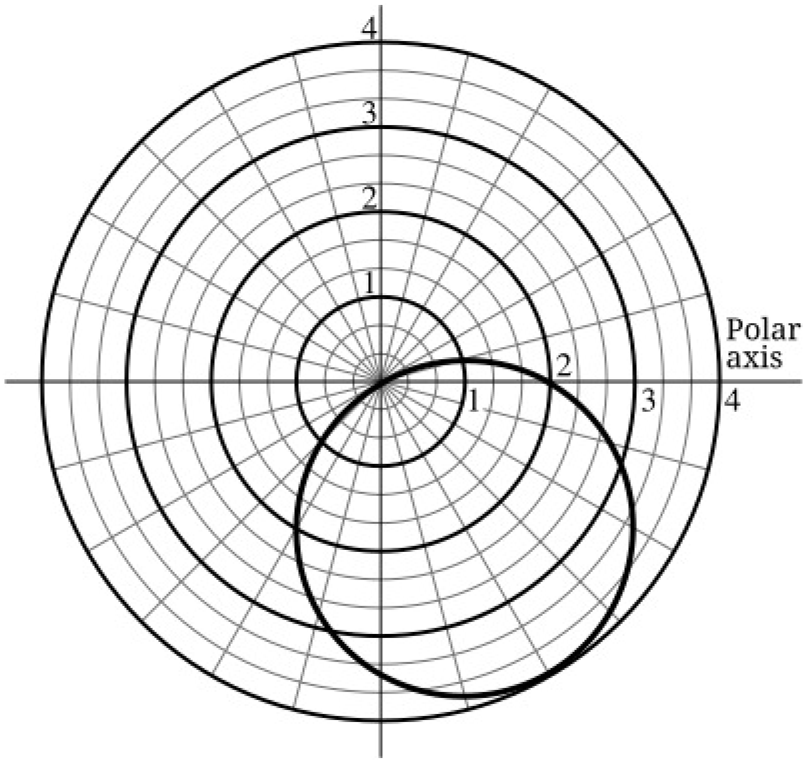
(D) $r = m(\theta)$, where $m(\theta) = \frac{1}{5}$ for $\pi \leq \theta \leq 2\pi$

Answer C

Correct. On this interval of θ, negative radius values produce points that correspond to the same semicircle with the same radius.

MCQ_Review_Unit_3

89.



The graph of the polar function $r = g(\theta)$, where $g(\theta) = 4\cos\left(\theta + \frac{\pi}{3}\right)$, is given in the polar coordinate system. Which of the following descriptions is true?

(A) Values of θ for $\frac{\pi}{2} < \theta < \pi$ correspond to the portion of the graph of $r = g(\theta)$ in Quadrant I.

(B) Values of θ for $\frac{\pi}{2} < \theta < \pi$ correspond to the portion of the graph of $r = g(\theta)$ in Quadrant II.

(C) Values of θ for $\frac{\pi}{2} < \theta < \pi$ correspond to the portion of the graph of $r = g(\theta)$ in Quadrant III.

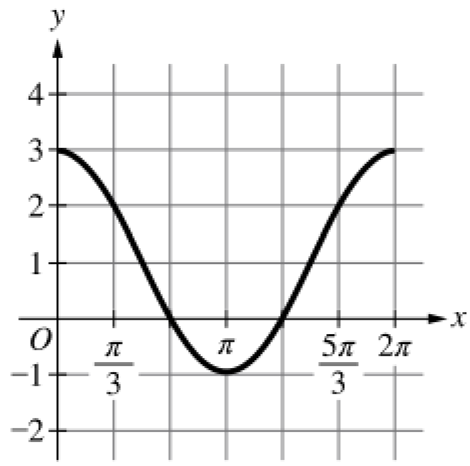
(D) Values of θ for $\frac{\pi}{2} < \theta < \pi$ correspond to the portion of the graph of $r = g(\theta)$ in Quadrant IV.

Answer D

Correct. By inspection of the points at $\theta = \frac{\pi}{2}$, $\theta = \pi$, and a value θ on the interval $\frac{\pi}{2} < \theta < \pi$, all of the points within the interval are in Quadrant IV.

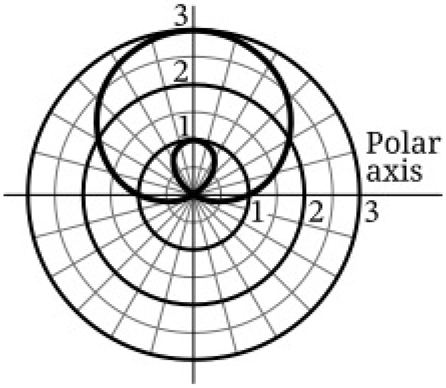
MCQ_Review_Unit_3

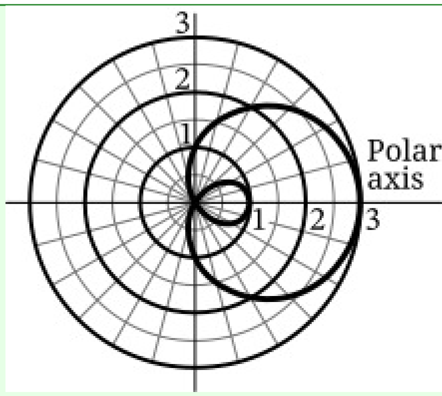
90.

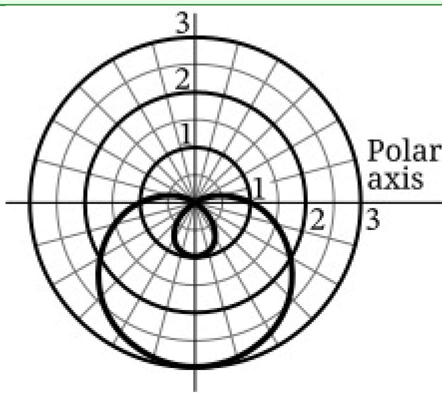


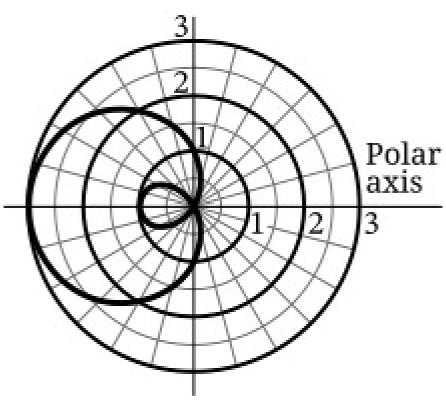
The graph of the function $y = f(x)$ is shown in the xy-plane. Which of the following is the graph of the polar function $r = f(\theta)$ in the polar coordinate system?

MCQ_Review_Unit_3

(A) 

(B) 

(C) 

(D) 

MCQ_Review_Unit_3

Answer B

Correct. The graph is the function $y = 2\cos x + 1$. This is the graph of the polar function $r = f(\theta)$, where $f(\theta) = 2\cos\theta + 1$. By considering the radius r at sample points (particularly at 0, $\frac{2\pi}{3}$, π, $\frac{4\pi}{3}$, and 2π) as well as increasing and decreasing intervals, it can be determined that this is the polar graph $r = f(\theta)$ that corresponds to $y = f(x)$.