MCQ_Review_Unit_2

1. The functions f and g are given by $f(x) = x^2 + 1$ and $g(x) = 4x - 1$. Which of the following is an expression for $f(g(x))$?

(A) $x^2 + 4x$

(B) $4x^2 + 3$

(C) $16x^2 - 8x + 2$ ✓

(D) $4x^3 - x^2 + 4x - 1$

Answer C

Correct. This is the result of correctly composing the functions as $(4x - 1)^2 + 1$ and rewriting the expression in an expanded standard form.

2. Two functions f and g are composed such that $f(g(x)) = 4\sqrt{x - 3} + 2$. Which of the following could be the expressions for $f(x)$ and $g(x)$?

(A) $f(x) = \sqrt{x}$ and $g(x) = 4x - 3$

(B) $f(x) = 4x - 3$ and $g(x) = \sqrt{x}$

(C) $f(x) = \sqrt{x - 3}$ and $g(x) = 4x + 2$

(D) $f(x) = 4x + 2$ and $g(x) = \sqrt{x - 3}$ ✓

Answer D

Correct. This is the result of decomposing the functions such that if $g(x)$ were substituted for x in every instance in the expression for $f(x)$, the result would be the given composition.

3. The functions f and g are given by $f(x) = \frac{1}{x}$ and $g(x) = \sqrt{x}$. What is the domain of the function h given by $h(x) = f(g(x))$?

(A) All real numbers

(B) All real numbers not equal to 0

(C) All real numbers greater than 0 ✓

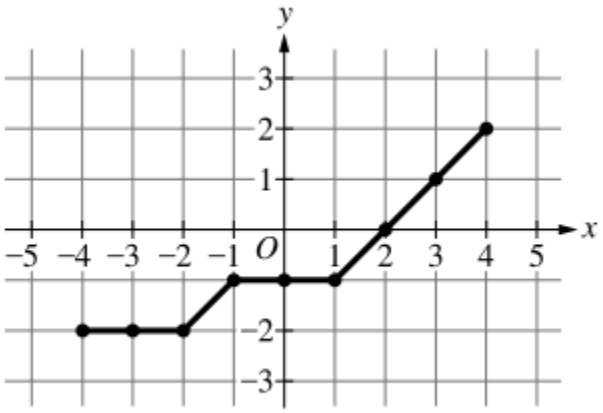
(D) All real numbers greater than or equal to 0

MCQ_Review_Unit_2

Answer C

Correct. The domain of g is all real numbers greater than or equal to 0. When the functions are composed to create function h, 0 is no longer in the domain because it results in division by 0.

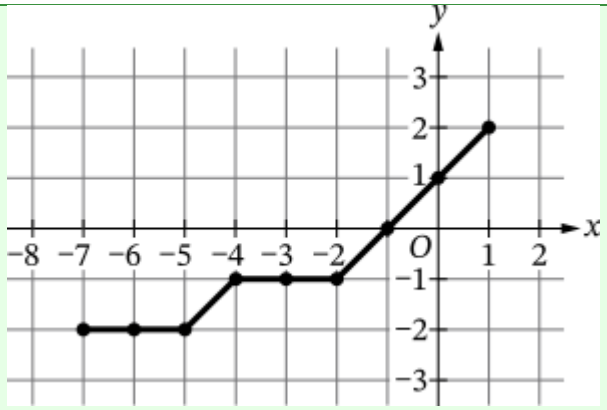
4.

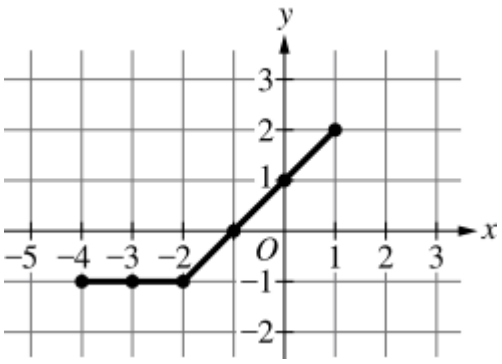


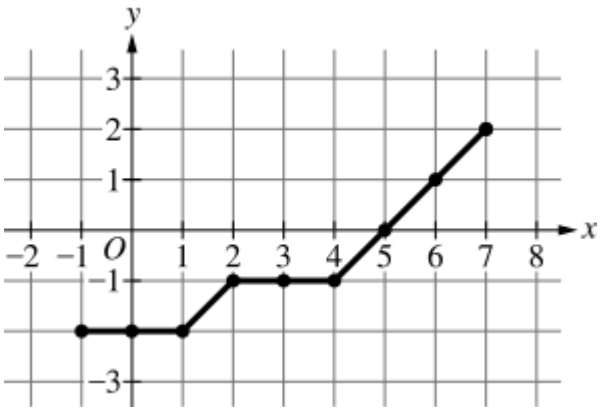
Graph of f

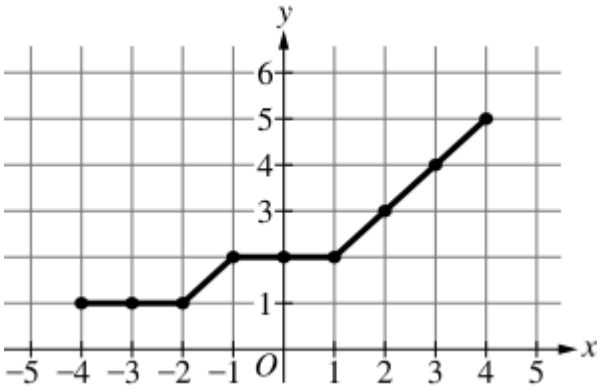
The piecewise-linear function f, defined on $-4 \leq x \leq 4$, is shown in the graph. The function g is given by $g(x) = x + 3$. Which of the following is the graph of $y = f(g(x))$?

MCQ_Review_Unit_2

(A) 

(B) 

(C) 

(D) 

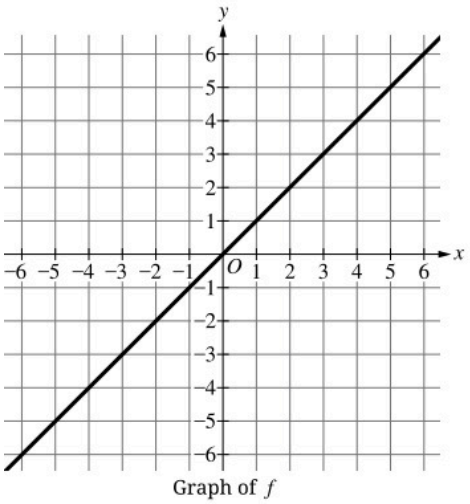
MCQ_Review_Unit_2

Answer A

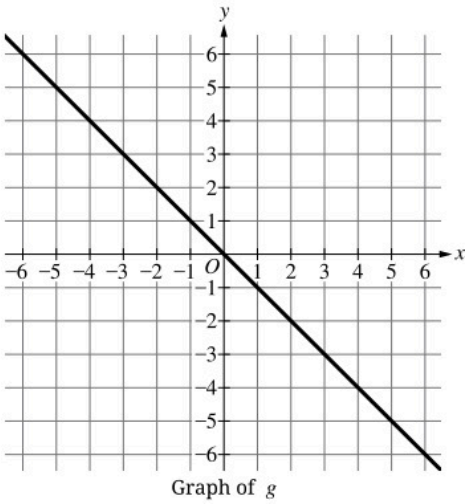
Correct. This is the result of translating f by 3 units left. Although the domain restriction of f is $-4 \leq x \leq 4$, the domain restriction of $f(g(x))$ is $-7 \leq x \leq 1$ because the composition is a horizontal translation of f.

MCQ_Review_Unit_2

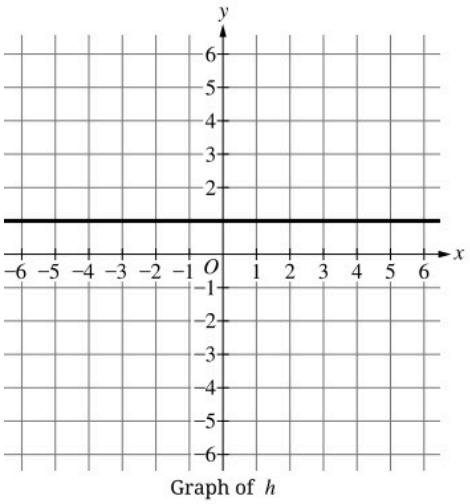
5.



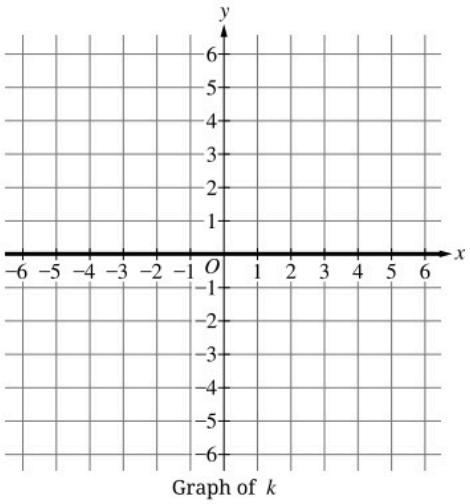
Graph of f



Graph of g



Graph of h



Graph of k

The function p (not shown) is a polynomial function of degree 3. The graphs of four functions f, g, h, and k are given.

The output values of p are the same as the output values of the composition function when p is composed with one of these functions as the input function. For which of the functions is this statement true?

(A) f ✓

(B) g

(C) h

(D) k

MCQ_Review_Unit_2

Answer A

Correct. Since f is the identity function $f(x) = x$ for composition, the composition will produce the same output values as the original function.

6.

x	-1	0	1
$f(x)$	0	1	-1

The table gives values of the function f for selected values of x. The function g is given by $g(x) = x^2$. Which of the following gives values of $g(f(x))$ for $x = -1$, $x = 0$, and $x = 1$?

(A)

x	-1	0	1
$g(f(x))$	-1	1	-1

(B)

x	-1	0	1
$g(f(x))$	0	0	-1

(C) ✓

x	-1	0	1
$g(f(x))$	0	1	1

(D)

x	-1	0	1
$g(f(x))$	1	1	0

MCQ_Review_Unit_2

Answer C

Correct. This is the result of taking each output value of f and using it as an input value for g.

7. Which of the following tables provides evidence that f is an exponential function if $y = f(x)$?

MCQ_Review_Unit_2

(A)

x	y
1	3
2	6
3	9
4	12

(B)

x	y
5	30
6	42
7	56
8	72

(C)

x	$\ln y$
1	1
2	3
3	9
4	27

MCQ_Review_Unit_2

(D)

x	$\ln y$
5	50
6	60
7	70
8	80

Answer D

Correct. Because both the values in the x and $\ln y$ columns form linear sequences, $y = f(x)$ is an exponential function.

8. The functions f and g are given by $f(x) = 4^{(5x-1)}$ and $g(x) = 8^{(x/4)}$. When solving the equation $f(x) = g(x)$, the functions can be rewritten in equivalent forms so that the equation can be solved without the use of technology. Which of the following are equivalent definitions of f and g that aid in solving $f(x) = g(x)$ without the use of technology?

(A) $f(x) = 2^{(\log_2 4 \cdot (5x-1))}$ and $g(x) = 2^{(\log_2 8 \cdot (x/4))}$

(B) $f(x) = 2^{(\log_2 8 \cdot (5x-1))}$ and $g(x) = 2^{(\log_2 4 \cdot (x/4))}$

(C) $f(x) = 4^{(\log_2 4 \cdot (5x-1))}$ and $g(x) = 8^{(\log_2 8 \cdot (x/4))}$

(D) $f(x) = 2 \cdot 4^{(\log_2 4 \cdot (5x-1))}$ and $g(x) = 8^{(\log_2 8 \cdot (x/4))}$

Answer A

Correct. By rewriting each expression with the same base 2 and a logarithm that equals an integer, the equation $f(x) = g(x)$ is more easily solved without technology.

9. The function f is given by $f(x) = 4 \cdot 2^{(x-3)}$. If the function g is the inverse of f, which of the following could define $g(x)$?

MCQ_Review_Unit_2

(A) $\log_8 x + 3$

(B) $\log_2(4x) + 3$

(C) $\log_2\left(\frac{x}{4}\right) + 3$ ✓

(D) $\log_2\left(\frac{x-3}{4}\right)$

Answer C

Correct. This expression for $g(x)$ can be found by step-by-step reversal of the operations of f.

10. Iodine-131 has a half-life of 8 days. In a particular sample, the amount of iodine-131 remaining after d days can be modeled by the function h given by $h(d) = A_0(0.5)^{(d/8)}$, where A_0 is the amount of iodine-131 in the sample at time $d = 0$. Which of the following functions k models the amount of iodine-131 remaining after t hours, where A_0 is the amount of iodine-131 in the sample at time $t = 0$? (There are 24 hours in a day, so $t = 24d$.)

(A) $k(t) = A_0(0.5)^{(t/24)}$

(B) $k(t) = A_0\left(0.5^{(1/24)}\right)^{(8t)}$

(C) $k(t) = A_0\left(0.5^{(24)}\right)^{(t/8)}$

(D) $k(t) = A_0\left(0.5^{(1/192)}\right)^{t}$ ✓

Answer D

Correct. To write the function h in an equivalent form in terms of t hours rather than d days, the exponent must be rewritten. Solving $t = 24d$ for d gives $d = \frac{t}{24}$. By substitution in the exponent of function h, $\frac{d}{8} = \frac{(t/24)}{8} = \frac{t}{192} = \left(\frac{1}{192}\right)t$.

11. The value, in millions of dollars, of transactions processed by an online payment platform is modeled by the function M. The value is expected to increase by 6.1% each quarter of a year. At time $t = 0$ years, 54 million dollars of transactions were processed. If t is measured in years, which of the following is an expression for $M(t)$? (Note: A quarter is one fourth of a year.)

(A) $54(0.061)^{(t/4)}$

(B) $54(0.061)^{(4t)}$

(C) $54(1.061)^{(t/4)}$

(D) $54(1.061)^{(4t)}$ ✓

MCQ_Review_Unit_2

Answer D

Correct. The function M is an exponential growth function and can be written in the form $M(t) = ab^t$. For $t = 0$, $M(t) = 54$ results in $ab^0 = 54$ and $a = 54$. Because there are 4 quarters in a year and t represents years, the exponent must be $4t$. Because of an increase of 6.1% each quarter, the base b is equal to 1.061. For exponential growth, $b > 1$.

12. The function g is a function of the form $g(x) = a \cdot b^x$, where $a \neq 0$ and $b > 0$. The function f is given by $f(x) = g(x) + 4$. Which of the following statements is true?

(A) The output values of both f and g are proportional over equal-length input-value intervals.

(B) The output values of f only, not g, are proportional over equal-length input-value intervals.

(C) The output values of g only, not f, are proportional over equal-length input-value intervals. ✓

(D) The output values of neither f nor g are proportional over equal-length input-value intervals.

Answer C

Correct. Because g is a function of the form $g(x) = a \cdot b^x$, where $a \neq 0$ and $b > 0$, g is exponential and the statement is true for g. However, because f is an additive transformation of g, each of the output values of f would need to change by a constant of -4 to be proportional over equal-length input-value intervals.

13. The value $(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 4.7)$ is the output value of an exponential function of the form $f(x) = a \cdot b^x$, where a and b are constants. Which of the following describes the function and input value that corresponds to this output value?

(A) The exponential function has an initial value of 1 and a base of $(2 \cdot 4.7)$, and the input value is 5.

(B) The exponential function has an initial value of 2 and a base of 2, and the input value is 4.7.

(C) The exponential function has an initial value of 4.7 and a base of 2, and the input value is 4.7.

(D) The exponential function has an initial value of 4.7 and a base of 2, and the input value is 5. ✓

Answer D

Correct. The common ratio, or base value, of 2 is repeatedly multiplied 5 times (the input value) to an initial value of 4.7.

MCQ_Review_Unit_2

14. The function f is given by $f(x) = 5 \cdot (0.7)^x$. Which of the following describes f ?

(A) The function f models exponential decay and $\lim_{x \to \infty} f(x) = 0$. ✓

(B) The function f models exponential decay and $\lim_{x \to \infty} f(x) = \infty$.

(C) The function f models exponential growth and $\lim_{x \to \infty} f(x) = 0$.

(D) The function f models exponential growth and $\lim_{x \to \infty} f(x) = \infty$.

Answer A

Correct. Because the common ratio is $0.7 < 1$ and the initial value is positive, this exponential function models decay. As the x-values increase without bound, the function values get closer to 0.

15. The function f is given by $f(x) = 2^x$, and the function g is given by $g(x) = \frac{f(x)}{8}$. For which of the following transformations is the graph of g the image of the graph of f ?

(A) A horizontal translation to the left 3 units

(B) A horizontal translation to the right 3 units ✓

(C) A vertical translation up $\frac{1}{8}$ unit

(D) A vertical translation down $\frac{1}{8}$ unit

Answer B

Correct. The function g can be written as $g(x) = \frac{2^x}{2^3} = 2^{(x-3)}$, which is a horizontal translation of the graph of f.

---

x	0	1	2	3	4
$f(x)$	$\frac{3}{4}$	$\frac{3}{2}$	3	6	12

The exponential function f is defined by $f(x) = ab^x$, where a and b are positive constants. The table gives values of $f(x)$ at selected values of x.

MCQ_Review_Unit_2

16. Which of the following statements is true?

(A) f demonstrates exponential decay because $a > 0$ and $0 < b < 1$.

(B) f demonstrates exponential decay because $a > 0$ and $b > 1$.

(C) f demonstrates exponential growth because $a > 0$ and $0 < b < 1$.

(D) f demonstrates exponential growth because $a > 0$ and $b > 1$. ✓

Answer D

Correct. The exponential function f for the values in the table is $f(x) = \frac{3}{4} \cdot 2^x$, where $a = \frac{3}{4}$ and $b = 2$.

17.

x	0	2	4	6
$f(x)$	3	48	768	12,288

The table gives values of the function f for selected values of x. Which of the following expressions could define $f(x)$?

(A) $2 + 4^x$

(B) $3 \cdot 4^x$ ✓

(C) $3 \cdot 16^x$

(D) $4 \cdot 3^x$

Answer B

Correct. This is the result of finding the common ratio of 4 and adjusting for the initial value.

MCQ_Review_Unit_2

18. In a certain town, the population in the year 2000 was about $30{,}000$. The population grows at a rate of 2.3% per year, and time is measured in years since 2000. Which of the following functions gives output values, in years since 2000, for input values of the town's population p ?

(A) $f(p) = 30{,}000 \cdot (1.023)^p$

(B) $g(p) = \log_{1.023}\left(\frac{p}{30{,}000}\right)$ ✓

(C) $h(p) = \frac{p - 30{,}000}{2.3}$

(D) $k(p) = 30{,}000 \cdot \log_{1.023} p$

Answer B

Correct. The exponential model is $f(x) = 30{,}000 \cdot (1.023)^x$, which gives population as a function of time x. This model is the inverse of f.

19. The function m is given by $m(x) = 36^{(x/2)}$. Which of the following expressions could also define $m(x)$?

(A) 6^x ✓

(B) $6 \cdot 6^x$

(C) 18^x

(D) $18 \cdot 36^x$

Answer A

Correct. Using the product property, $36^{(x/2)} = \left(36^{(1/2)}\right)^x$. The fractional exponent $\frac{1}{2}$ indicates the square root of the value, or $36^{(1/2)} = 6$.

20. The function h is given by $h(x) = 5 \cdot 3^{(-x/2)}$. What is the value of $h(1)$?

(A) $-5\sqrt{3}$

(B) $\frac{1}{\sqrt{15}}$

(C) $\frac{5}{9}$

(D) $\frac{5}{\sqrt{3}}$ ✓

MCQ_Review_Unit_2

Answer D

Correct. This is the result of correctly applying the negative exponent property to get $h(1) = 5 \cdot 3^{(-1/2)} = \frac{5}{3^{(1/2)}} = \frac{5}{\sqrt{3}}$.

21. The function k is given by $k(x) = 9^x$. Which of the following expressions also defines $k(x)$?

(A) $2^{(3x)}$

(B) $3^{(2x)}$ ✓

(C) $3^{(3x)}$

(D) $3^{(x/2)}$

Answer B

Correct. Using the power property for exponents and the fact that $9 = 3^2$, $k(x)$ can be expressed in this equivalent form.

22. The function f is given by $f(x) = 3^x$. The function g is given by $g(x) = (f(x))^b$, where $b < 0$. Which of the following describes the relationship between the graphs of f and g ?

(A) The graph of g is a combination of a horizontal dilation of the graph of f and a reflection over the x -axis.

(B) The graph of g is a combination of a horizontal dilation of the graph of f and a reflection over the y -axis. ✓

(C) The graph of g is a combination of a vertical dilation of the graph of f and a reflection over the x-axis.

(D) The graph of g is a combination of a vertical dilation of the graph of f and a reflection over the y-axis.

Answer B

Correct. The power property implies a horizontal dilation. Because $b < 0$ and it impacts x, there is also a reflection over the y-axis.

23. Water hyacinth is an invasive plant species found in many lakes that typically grows at a rate of 7% per day. As part of a study, a scientist introduces a 150-gram sample of water hyacinth into a testing pool. Which of the following functions gives the amount of water hyacinth in the testing pool t weeks after the sample is introduced? (Note: 1 week is 7 days.)

MCQ_Review_Unit_2

(A) $f(t) = 150\left(1 + 0.07^{(1/7)}\right)^t$

(B) $g(t) = 150\left(1.07^{(1/7)}\right)^t$

(C) $h(t) = 150\left(1 + 0.07^{(7)}\right)^t$

(D) $k(t) = 150\left(1.07^{(7)}\right)^t$ ✓

Answer D

Correct. Each week the growth factor of $(100 + 7)\%$ percent must be applied 7 times.

24. The function f is given by $f(x) = x^2 + 1$, and the function g is given by $g(x) = \frac{(x-3)}{x}$. Which of the following is an expression for $f(g(x))$?

(A) $\frac{x^3-3x^2+x-3}{x}$

(B) $\frac{x^2-2}{x^2+1}$

(C) $\frac{x^2-6x+9}{x^2} + 1$ ✓

(D) $\frac{x^2-8}{x^2}$

Answer C

Correct. The composition of functions $f(g(x))$ can be constructed by substituting $g(x)$ for every instance of x in f. $f(g(x)) = \left(\frac{x-3}{x}\right)^2 + 1 = \frac{(x-3)^2}{x^2} + 1$.

MCQ_Review_Unit_2

25.

x	$g(x)$
-2	4
0	$\frac{1}{2}$
3	-2
4	3
36	9

The table gives values of the function g for selected values of x. The function f is given by $f(x) = 3^x + x^2$. What is the value of $f(g(3))$?

(A) -72

(B) $\frac{37}{9}$ ✓

(C) 9

(D) 97

Answer B

Correct. Using the table of values, $g(3) = -2$. The output value of $g(3)$ becomes the input value for function f. Solving for $f(-2)$, $f(-2) = 3^{(-2)} + (-2)^2 = \frac{1}{9} + 4 = \frac{37}{4}$.

26. The functions f and g are given by $f(x) = 2^x$ and $g(x) = 2^x \cdot 2^a$, where $a > 0$. Which of the following describes the relationship between the graph of f and the graph of g ?

MCQ_Review_Unit_2

(A) The graph of g is a vertical translation of the graph of f by a units.

(B) The graph of g is a horizontal translation of the graph of f by a units.

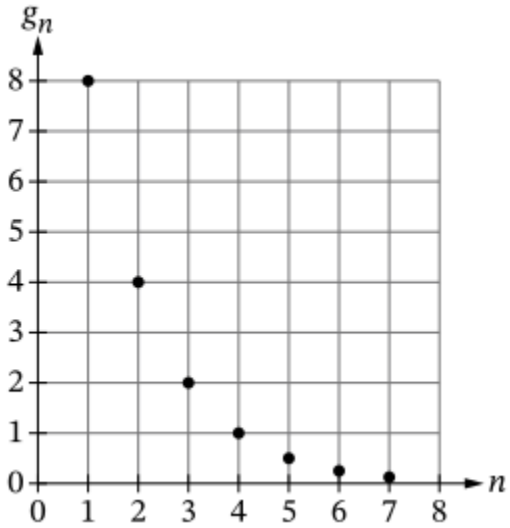
(C) The graph of g is a vertical translation of the graph of f by $-a$ units.

(D) The graph of g is a horizontal translation of the graph of f by $-a$ units. ✓

Answer D

Correct. By the product property, $g(x) = 2^{(x+a)} = f(x+a)$, which is a horizontal translation of the graph of $f(x)$ by $-a$ units.

27.



Values of the terms of a geometric sequence g_n are graphed in the figure. Which of the following is an expression for the nth term of the geometric sequence?

(A) $g_n = 4\left(\frac{1}{2}\right)^{(n-2)}$ ✓

(B) $g_n = 8(2)^{(n-1)}$

(C) $g_n = 8\left(\frac{1}{2}\right)^{n}$

(D) $g_n = 16\left(\frac{1}{2}\right)^{(n-1)}$

Answer A

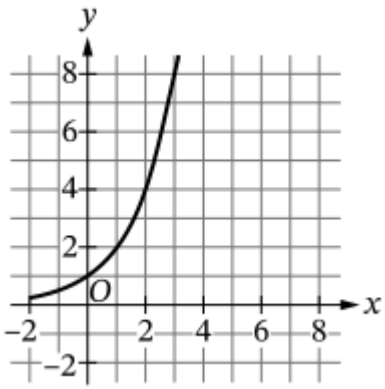
Correct. In this case, the initial value of the geometric sequence is $g_1 = 8$. The value of the terms of the

geometric sequence g_n are such that the common ratio is $r = \frac{1}{2}$ and $g_2 = 4$. Using the general term of a geometric sequence $g_n = g_k r^{(n-k)}$, where g_k is the $k\text{th}$ term, the geometric sequence for the graph of values can be defined as $g_n = 4\left(\frac{1}{2}\right)^{(n-2)}$.

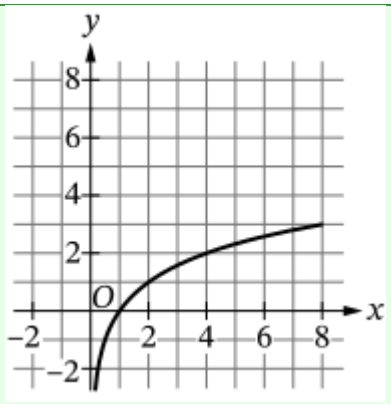
28. The function g has the property that for each time the input values double, the output values increase by 1. Which of the following could be the graph of $y = g(x)$ in the xy-plane?

MCQ_Review_Unit_2

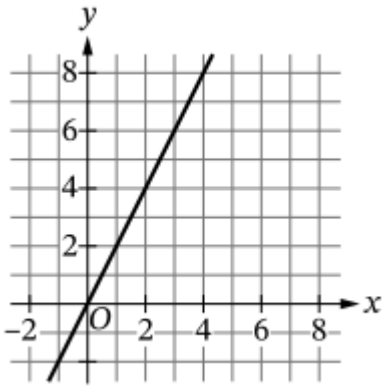
(A)



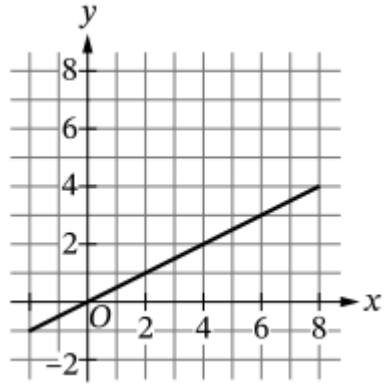
(B) ✓



(C)



(D)



MCQ_Review_Unit_2

Answer B

Correct. Function g is a logarithmic function that could be modeled graphically by $y = \log_2 x$. This function has the desired property.

29. The function f is defined by $f(x) = \sqrt{4 - x^2}$ for $-2 \le x \le 0$. Which of the following expressions defines $f^{-1}(x)$?

(A) $-\sqrt{4 - x^2}$ for $-2 \le x \le 0$

(B) $\sqrt{4 - x^2}$ for $-2 \le x \le 0$

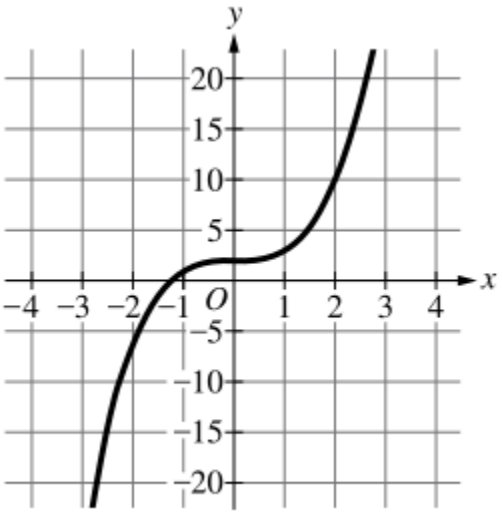
(C) $-\sqrt{4 - x^2}$ for $0 \le x \le 2$ ✓

(D) $\sqrt{4 - x^2}$ for $0 \le x \le 2$

Answer C

Correct. This is the result of reversing the roles of x and y in the equation $y = \sqrt{4 - x^2}$ and solving for the inverse function. The domain of the inverse function f^{-1} is the range of the original function f. The range of the inverse function f^{-1} is the domain of the original function f.

30.

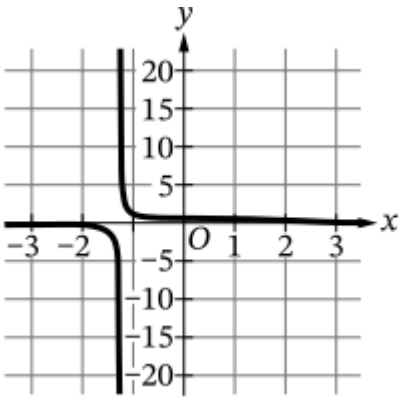


Graph of f

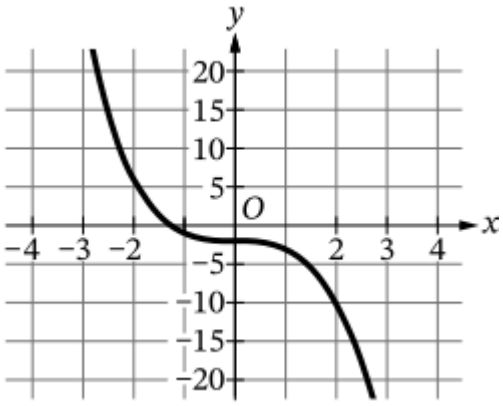
The graph of the function $y = f(x)$ is given. Which of the following is the graph of $y = f^{-1}(x)$?

MCQ_Review_Unit_2

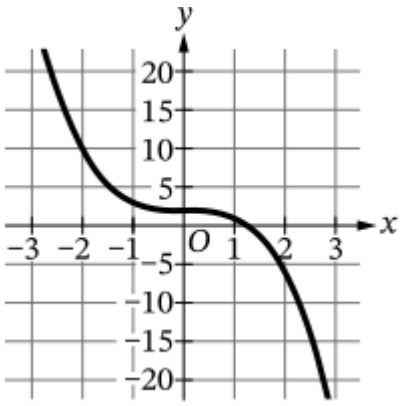
(A)



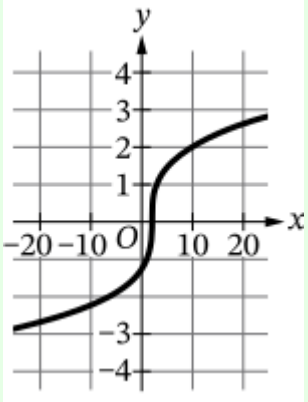
(B)



(C)



(D)



MCQ_Review_Unit_2

Answer D

Correct. This graph is a reflection of the graph of f over the identity function $h(x) = x$.

31. The function g is given by $g(x) = \frac{4x+6}{5}$. Which of the following defines $g^{-1}(x)$?

(A) $\frac{5}{4x+6}$

(B) $\frac{5x+6}{4}$

(C) $\frac{5x}{4} - 6$

(D) $\frac{5x-6}{4}$ ✓

Answer D

Correct. This is the result of reversing the operations of g with multiplication by 5 first, then subtraction of 6, followed by division of the entire expression by 4.

32. The function f is defined by $f(x) = 4x^2 + 3$ for $x \geq 0$. Which of the following expressions defines the inverse function of f ?

(A) $f^{-1}(x) = \frac{x^2}{4} - 3$ for $x \geq 0$

(B) $f^{-1}(x) = \sqrt{\frac{x}{4}} - 3$ for $x \geq 0$

(C) $f^{-1}(x) = \sqrt{\frac{x-3}{4}}$ for $x \geq 3$ ✓

(D) $f^{-1}(x) = \frac{\sqrt{x-3}}{4}$ for $x \geq 3$

Answer C

Correct. This is the result of reversing the roles of x and y in the equation $y = 4x^2 + 3$and then solving for y. Note that the values in the domain of f^{-1} correspond to the values in the range of f.

33. A water tank is leaking water from a crack in its base. The amount of water, in hundreds of gallons, remaining in the tank t hours after the crack formed can be modeled by W, a decreasing function of time t. Which of the following gives a verbal representation of the function W^{-1}, the inverse function of W ?

MCQ_Review_Unit_2

(A) W^{-1} is an increasing function of the amount of time after the crack formed.

(B) W^{-1} is a decreasing function of the amount of time after the crack formed.

(C) W^{-1} is an increasing function of the amount of water in the tank.

(D) W^{-1} is a decreasing function of the amount of water in the tank. ✓

Answer D

Correct. Because W maps times to amounts of water, the inverse function maps amounts of water to times. Because W is decreasing, as one variable increases, the other decreases. This relationship does not change for the inverse function.

34. The function f is given by $f(x) = \log_2(\log_3 x)$. Which of the following is an expression for $f^{-1}(x)$?

(A) $2^{(3^x)}$

(B) $3^{(2^x)}$ ✓

(C) $2 \cdot 3^x$

(D) $3 \cdot 2^x$

Answer B

Correct. When the function and its inverse function are composed, the result is the identity function for composition $y = x$. Working from the inside out, $\log_2\left(\log_3\left(3^{(2^x)}\right)\right) = \log_2(2^x) = x$.

35.

x	1	2	3	4
$f(x)$	2	4	8	16

The table gives values of the function f for selected values of x. Which of the following is a verbal representation of $f^{-1}(x)$, the inverse function of f ?

MCQ_Review_Unit_2

(A) $f^{-1}(x)$ is logarithmic with input values increasing by 1 every time output values double.

(B) $f^{-1}(x)$ is logarithmic with output values increasing by 1 every time input values double. ✓

(C) $f^{-1}(x)$ is exponential with input values increasing by 1 every time output values double.

(D) $f^{-1}(x)$ is exponential with output values increasing by 1 every time input values double.

Answer B

Correct. The data in the table suggest an exponential model in which every time input values increase by 1, output values double. This is the inverse of that relationship.

36. The exponential function g is given by $g(x) = 5^x$. Which of the following expressions defines $g^{-1}(x)$?

(A) $\log_5 x$ ✓

(B) $\log_x 5$

(C) $\sqrt[5]{x}$

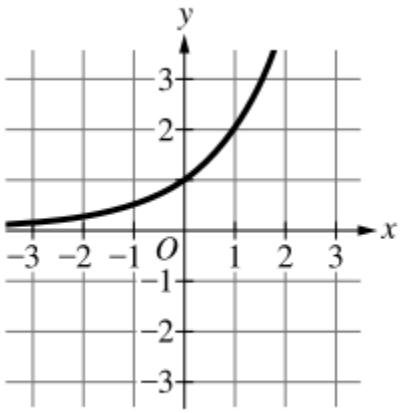
(D) $\sqrt[x]{5}$

Answer A

Correct. The inverse of an exponential function of base 5 is the logarithmic function of base 5.

MCQ_Review_Unit_2

37.



Graph of f

The graph of the exponential function f is given. Which of the following could be a table of values for the inverse function of f ?

MCQ_Review_Unit_2

(A)

x	5	2
$f^{-1}(x)$	32	4

(B)

x	5	2
$f^{-1}(x)$	25	4

(C)

x	25	4
$f^{-1}(x)$	5	2

(D)

x	32	4
$f^{-1}(x)$	5	2

MCQ_Review_Unit_2

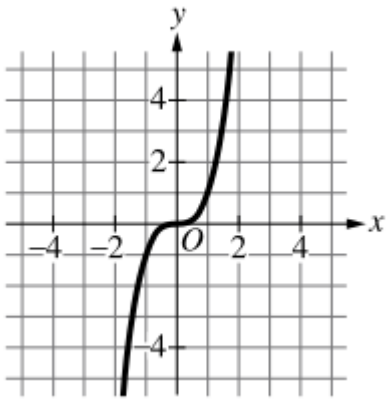
Answer D

Correct. The original function f is $f(x) = 2^x$. For the inverse function, the input values will be the power of 2 indicated by the output value of the inverse function.

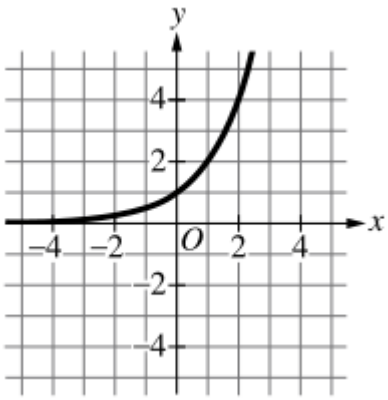
38. The function f is given by $f(x) = \log_3 x$. Which of the following could be the graph of $y = f^{-1}(x)$?

MCQ_Review_Unit_2

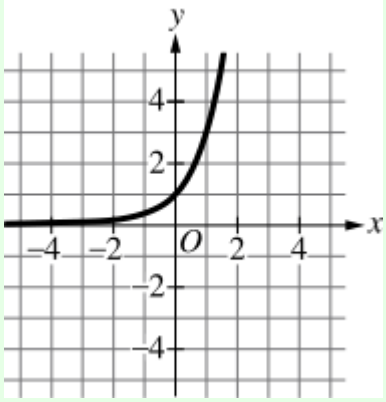
(A)



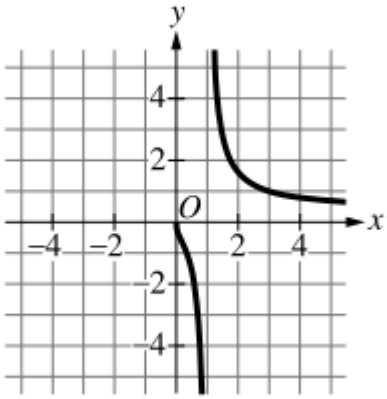
(B)



(C) ✓



(D)



MCQ_Review_Unit_2

Answer C

Correct. The inverse function is $f^{-1}(x) = 3^x$.

39. The function f is an increasing function such that every time the output values of the function f increase by 1, the corresponding input values multiply by 4. Which of the following could define $f(x)$?

(A) x^4

(B) 4^x

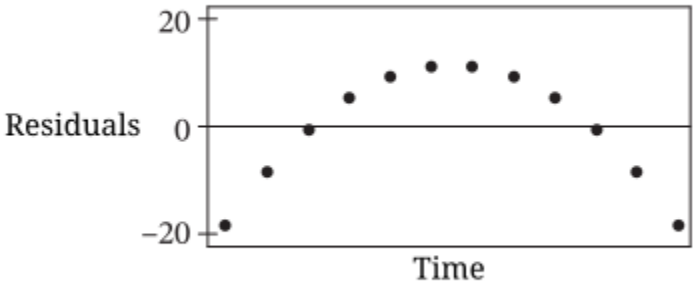
(C) $\sqrt[4]{x}$

(D) $\log_4 x$ ✓

Answer D

Correct. General-form logarithmic functions like $f(x) = \log_4 x$ have the property that their input values change proportionately as their output values increase in equal-length intervals. For the function described, 4 is the base for the logarithmic function.

40.



A food vendor developed a new sandwich type for sale. The vendor made estimates about the sales of the new sandwich type over time. A linear regression was used to develop a model for the sales over time. The figure shows a graph of the residuals of the linear regression. Which of the following statements about the linear regression is true?

MCQ_Review_Unit_2

(A) The linear model is not appropriate, because there is a clear pattern in the graph of the residuals. ✓

(B) The linear model is not appropriate, because the graph of the residuals has more points above 0 than below 0.

(C) The linear model is appropriate, because there is a clear pattern in the graph of the residuals.

(D) The linear model is appropriate, because the positive residual farthest from 0 and the negative residual farthest from 0 are about the same distance, although more points are above 0 than below 0.

Answer A

Correct. A model is justified as appropriate for a data set if the graph of the residuals appear without pattern.

41. The range of function f is the positive real numbers. The function g is given by $g(x) = \ln(f(x))$. Solutions to which of the following equations are useful in solving $g(x) = 2$?

(A) $f(x) = 2$

(B) $f(x) = e^2$ ✓

(C) $f(x) = 10^2$

(D) $f(x) = \frac{2}{\ln x}$

Answer B

Correct. Because $g(x) = 2$, $\ln(f(x)) = 2$. By the definition of a logarithm, this equation can be rewritten as $e^2 = f(x)$.

42. To solve the equation $\log_8(x-3) + \log_8(x+4) = 1$, one method is to apply the properties of logarithms to write a new equation that can be used to identify possible solutions. Of the following, which is such an equation?

(A) $2x + 1 = 8$

(B) $\frac{x-3}{x+4} = 8$

(C) $x^2 - 12 = 8$

(D) $x^2 + x - 12 = 8$ ✓

MCQ_Review_Unit_2

Answer D

Correct. Using the product property for logarithms, the given equation has the same solutions as $\log_8((x-3)(x+4)) = 1$. Using the definition of a logarithm, this helpful quadratic equation emerges: $8^1 = (x-3)(x+4) = x^2 + x - 12$.

43. Which of the following is the inverse of the function f given by $f(x) = 4\log_2(x+3) - 1$?

(A) $g(x) = \frac{1}{4}(2^x + 1) - 3$

(B) $g(x) = \frac{1}{4} \cdot 2^{(x+1)} - 3$

(C) $g(x) = 2^{(\frac{x}{4}+1)} - 3$

(D) $g(x) = 2^{(\frac{x+1}{4})} - 3$ ✓

Answer D

Correct. This is the result of reversing the operations of the logarithmic function in the appropriate order to craft its inverse exponential function.

44. What are all values of x for which $\ln\left(x^3\right) - \ln x = 4$?

(A) $x = -2$ and $x = 2$

(B) $x = -e^2$ and $x = e^2$

(C) $x = e^2$ only ✓

(D) $x = e^4$

Answer C

Correct. The natural log side of the equation can be rewritten in an equivalent form as $\ln\left(\frac{x^3}{x}\right) = \ln\left(x^2\right)$, resulting in the equation $\ln\left(x^2\right) = 4$. Writing as an exponential equation gives $e^4 = x^2$ and solving for x results in $x = e^2$ and $x = -e^2$. However, $x = -e^2$ is not in the domain of the original equation because the value of $x = -e^2$ is negative.

MCQ_Review_Unit_2

45. An equation involves the expression $\log_9(27^x)$, which is equivalent to a rational multiple of x. By rewriting the expression in an equivalent form, the value of the rational number can be determined without use of a calculator or complicated calculations. Which of the following is an equivalent expression that satisfies this requirement?

(A) $x \ln\left(\frac{27}{9}\right)$

(B) $x \log_3\left(\frac{27}{9}\right)$

(C) $\frac{x \ln 27}{\ln 9}$

(D) $\frac{x \log_3 27}{\log_3 9}$ ✓

Answer D

Correct. Recognizing that both 27 and 9 are powers of 3 allows the change of base and power properties for logarithms to be used. $\log_9(27^x) = \frac{x \log_3 27}{\log_3 9}$, which is equivalent to $\frac{3}{2}x$. The rational number is $\frac{3}{2}$.

46. If $m = \log_3 81$, which of the following is also true?

(A) $3m = 81$

(B) $3^m = 81$ ✓

(C) $\sqrt[3]{m} = 81$

(D) $\sqrt[3]{81} = m$

Answer B

Correct. This is the result of correctly translating an equation involving a logarithmic expression to one involving an exponential expression.

47. The sales of a new product, in items per month, is modeled by the expression $225 + 500 \log_{10}(15t + 10)$, where t represents the time since the product became available for purchase, in months. What is the number of items sold per month for time $t = 6$?

(A) 725

(B) 1225 ✓

(C) 1700

(D) 5225

MCQ_Review_Unit_2

Answer B

Correct. Evaluating the expression for $t = 6$ results in $225 + 500 \log_{10}(15 \cdot 6 + 10)$, or $225 + 500 \log_{10}(100)$. Because $\log_{10}(100) = 2$, the expression is equal to $225 + 500 \cdot 2$.

48. The function f is given by $f(x) = 2 \log_5 x$. Which of the following describes f ?

(A) f is an increasing function that increases at an increasing rate.

(B) f is an increasing function that increases at a decreasing rate. ✓

(C) f is a decreasing function that decreases at an increasing rate.

(D) f is a decreasing function that decreases at a decreasing rate.

Answer B

Correct. This logarithmic function is increasing, and the graph of f is concave down. Therefore, the rate of change is decreasing.

49. Which of the following could describe a single logarithmic function f ?

(A) $\lim_{x \to 0^+} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = -\infty$

(B) $\lim_{x \to 0^+} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = k$, where k is a positive constant

(C) $\lim_{x \to 0^+} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = 0$

(D) $\lim_{x \to 0^+} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = -\infty$ ✓

Answer D

Correct. A logarithmic function can increase without bound asymptotically at $x = 0$ and decrease without bound as x increases without bound. An example is the function $f(x) = -\log_{10} x$.

50. The logarithmic function f is defined by $f(x) = \log_3 x$ on a domain of f is $0 < x \leq 9$. Which of the following is true of f ?

MCQ_Review_Unit_2

(A) f has both a maximum and a minimum value.

(B) f has a maximum value, but no minimum value. ✓

(C) f has a minimum value, but no maximum value.

(D) f has neither a minimum value nor a maximum value.

Answer B

Correct. Because $\lim_{x \to 0^+} f(x) = -\infty$, the function f has no minimum value. However, because f has a restricted domain and is an increasing function, f has a maximum value of $\log_3 9 = 2$ at the right endpoint domain value of $x = 9$.

51. The function f is given by $f(x) = 9 \cdot 25^x$. Which of the following is an equivalent form for $f(x)$?

(A) $f(x) = 3 \cdot 5^{(x/2)}$

(B) $f(x) = 3 \cdot 5^{(2x)}$

(C) $f(x) = 9 \cdot 5^{(x/2)}$

(D) $f(x) = 9 \cdot 5^{(2x)}$ ✓

Answer D

Correct. The exponential function f can be rewritten in this form as a result of the power property for exponents because $5^2 = 25$.

52. Consider the functions f and g given by $f(x) = \log_{10}(x - 1) + \log_{10}(x + 3)$ and $g(x) = \log_{10}(x + 9)$. In the xy-plane, what are all x-coordinates of the points of intersection of the graphs of f and g ?

(A) $x = 3$ only ✓

(B) $x = 7$

(C) $x = -4$ and $x = 3$

(D) $x = -7$ and $x = -4$

MCQ_Review_Unit_2

Answer A

Correct. Using properties of logarithms, $f(x) = \log_{10}((x-1)(x+3)) = \log_{10}\left(x^2+2x-3\right)$. Set $f(x) = g(x)$ to find the x-coordinates of the points of intersection of the two graphs. $\log_{10}\left(x^2+2x-3\right) = \log_{10}(x+9)$ and $x^2+2x-3 = x+9 \rightarrow x^2+x-12 = 0$. Solving for x, $(x+4)(x-3) = 0$ yields $x = -4$ or $x = 3$. Checking those values in the original functions f and g results in $x = -4$ as an extraneous solution because $x = -4$ is not in the domain of f.

53. The function g is given by $g(x) = \ln(3x+1) - \ln\left(x^2+x-2\right)$. What are all values of x for which $g(x) < 0$?

(A) $(-\infty, -1)$ and $(3, \infty)$

(B) $(-1,3)$

(C) $(1,3)$ only

(D) $(3, \infty)$ only ✓

Answer D

Correct. Solving the inequality $\ln(3x+1) - \ln\left(x^2+x-2\right) < 0$ leads to $3x+1 < x^2+x-2$, whose solutions are $(-\infty, -1)$ and $(3, \infty)$. However, only $(3, \infty)$ are in the domain of g.

54. A decibel (dB) is a unit of measure for loudness of sound. The decibel scale is based in sound intensity N, in watts per square meter. A decibel value is given by the function d, where $d(N) = 10\log_{10}\left(\frac{N}{10^{-12}}\right)$. Which of the following gives all intensities N, in watts per square meter, for which the decibel value is greater than 140 decibels?

(A) $N > 14 \cdot 10^{-12}$

(B) $N > 2$

(C) $N > 100$ ✓

(D) $N > 10^{26}$

MCQ_Review_Unit_2

Answer C

Correct. Solving the inequality $10 \log_{10}\left(\frac{N}{10^{-12}}\right) > 140$ using properties of logarithms leads to $\log_{10}\left(N \cdot 10^{12}\right) = \log_{10} N + 12 \log_{10} 10 > 14$. This results in $\log_{10} N + 12 > 14$ and $\log_{10} N > 2$. Therefore, $N > 10^2$.

55. The function f is given by $f(x) = \log_2 x$. Which of the following is equivalent to $f(7)$?

(A) $\log_{10}\left(\frac{7}{2}\right)$

(B) $\frac{\log_{10} 2}{\log_{10} 7}$

(C) $\frac{\log_7 10}{\log_2 10}$

(D) $\frac{\log_3 7}{\log_3 2}$ ✓

Answer D

Correct. This expression is equivalent to $\log_2 7$ by the change of base property for logarithms: $\log_b x = \frac{\log_a x}{\log_a b}$, where $a > 0$ and $a \neq 1$.

56. The function g is given by $g(x) = \log_7 x$, and the function h is given by $h(x) = \log_{49} x$. Which of the following describes the relationships between g and h ?

(A) For equal input values, the output values of h are half the output values of g. ✓

(B) For equal input values, the output values of h are twice the output values of g.

(C) For equal input values, the output values of h are the square of the output values of g.

(D) For equal input values, the output values of h are the square root of the output values of g.

Answer A

Correct. By the change of base property for logarithms, $h(x) = \log_{49} x = \frac{\log_7 x}{\log_7 49} = \frac{g(x)}{2}$.

57. The function h is given by $h(x) = \log_3 x$. Which of the following is equivalent to the expression $2 \cdot h(w) + h(p)$, where w and p are values in the domain of h ?

MCQ_Review_Unit_2

(A) $\log_3\left((wp)^2\right)$

(B) $(\log_3 w)^2 \cdot (\log_3 p)$

(C) $\log_3\left(w^2 p\right)$ ✓

(D) $\log_3(2wp)$

Answer C

Correct. Using both the power and the product properties for logarithms, the expression can be rewritten in an equivalent form: $2 \cdot h(w) + h(p) = 2\log_3 w + \log_3 p$. This is equivalent to $\log_3\left(w^2\right) + \log_3 p = \log_3\left(w^2 p\right)$.

58. The function f is given by $f(x) = \log_{10} x$. The function g is given by $g(x) = \log_{10}\left(x^3\right)$. Which of the following describes a transformation for which the graph of g is the image of the graph of f ?

(A) A vertical dilation by a factor of 3 ✓

(B) A vertical dilation by a factor of $\frac{1}{3}$

(C) A horizontal dilation by a factor of 3

(D) A horizontal dilation by a factor of $\frac{1}{3}$

Answer A

Correct. By the power property for logarithms, $\log_{10}\left(x^3\right) = 3\log_{10} x = 3f(x)$, which results in a vertical dilation of f by a factor of 3.

59. The function f is given by $f(x) = \ln x$. Which of the following describes input values for which the output values of f are integers?

(A) Integer powers of e ✓

(B) Integer powers of 10

(C) Integers raised to the power e

(D) Integers raised to the power 10

MCQ_Review_Unit_2

Answer A

Correct. Because $\ln x$ is how the natural logarithm with base e is written, integer powers of e will result in integer outputs for the function. For example, $\ln e^1 = 1$, $\ln e^4 = 4$, and $\ln e^{10} = 10$.

60. The function g is given by $g(x) = a \log_b c$, where a, b, and c are positive integers. Which of the following is an equivalent representation of $g(x)$?

(A) $\log_b(c^a)$ ✓

(B) $(\log_b c)^a$

(C) $\log_b\left(c^{(1/a)}\right)$

(D) $a \log_{10} b + a \log_{10} c$

Answer A

Correct. This expression is equivalent to $g(x)$ by the power property for logarithms.

61. The function f is logarithmic, and the points $(2,1)$ and $(4,2)$ are on the graph of f in the xy-plane. Which of the following could define $f(x)$?

(A) $\log_4 x$

(B) $2 \log_2 x$

(C) $2 \log_4 x$ ✓

(D) $\log_4(x+2)$

Answer C

Correct. Both points are on the graph of this function.

62. The initial population size of an animal species is measured to be 2000. The population doubles every 8 years. Which of the following functions gives the time, in years, as an output value, and a certain number x for the population size as an input value?

MCQ_Review_Unit_2

(A) $f(x) = \frac{1}{8}\log_2\left(\frac{x}{2000}\right)$

(B) $g(x) = \log_2\left(\frac{8x}{2000}\right)$

(C) $h(x) = 8\log_2\left(\frac{x}{2000}\right)$ ✓

(D) $k(x) = 2000\log_8 x$

Answer C

Correct. An exponential model for the population is $y = 2000 \cdot 2^{(x/8)}$. By using the reverse operations, the inverse logarithmic model for time, in years, can be found.

63. Let x and y be positive constants. Which of the following is equivalent to $2\ln x - 3\ln y$?

(A) $\ln\left(\frac{x^2}{y^3}\right)$ ✓

(B) $\ln\left(x^2y^3\right)$

(C) $\ln(2x - 3y)$

(D) $\ln\left(\frac{2x}{3y}\right)$

Answer A

Correct. The power and product properties for logarithms have both been appropriately applied to rewrite the logarithmic expression in equivalent form. $2\ln x - 3\ln y = \ln\left(x^2\right) + \ln\left(y^{-3}\right) = \ln\left(x^2y^{-3}\right)$.

64. Let k, w, and z be positive constants. Which of the following is equivalent to $\log_{10}\left(\frac{kz}{w^2}\right)$?

(A) $\log_{10}(k + z) - \log_{10}(2w)$

(B) $\log_{10} k + \log_{10} z - 2\log_{10} w$ ✓

(C) $\log_{10} k + \log_{10} z - \frac{1}{2}\log_{10} w$

(D) $\log_{10} k - \log_{10} z + 2\log_{10} w$

Answer B

Correct. This is the result of using the product property and the power property for logarithms to rewrite

the logarithmic expression in an equivalent form. $\log_{10}\left(\frac{kz}{w^2}\right) = \log_{10}(kz) - \log_{10}\left(w^2\right)$, which is equivalent to $\log_{10} k + \log_{10} z - 2\log_{10} w$.

65. The function f is given by $f(x) = a \cdot c^x$, where $a > 0$ and $c > 1$. Which of the following is true about the values of constants m and b in the equation $\ln(f(x)) = mx + b$?

(A) $m > 0$ because $\ln c > 0$; b can be any real number because $\ln a$ can be any real number. ✓

(B) $m > 0$ because $\ln c > 0$; $b > 0$ because $\ln a > 0$.

(C) m can be any real number because $\ln c$ can be any real number; b can be any real number because $\ln a$ can be any real number.

(D) m can be any real number because $\ln c$ can be any real number; $b > 0$ because $\ln a > 0$.

Answer A

Correct. The conclusions and rationales about all values are true.

66. In a semi-log plot, which of the following pairs of functions appear linear as parallel lines?

(A) $f(x) = 2x$ and $g(x) = 2x + 3$

(B) $f(x) = x^2$ and $g(x) = 3x^2$

(C) $f(x) = 2^x$ and $g(x) = 3 \cdot 2^x$ ✓

(D) $f(x) = \ln(2x)$ and $g(x) = 3\ln(2x)$

Answer C

Correct. Exponential functions appear linear in a semi-log plot. The graphs of these two functions in a semi-log plot appear as parallel lines with slope $\log_n 2$ and y-intercepts 0 and $\log_n 3$, respectively, depending on base n of the logarithmic scale.

MCQ_Review_Unit_2

67.

x	5	6	7	8
$\ln y$	3	6	9	12

The table gives ordered pairs $(x, \ln y)$. For the function $y = f(x)$, which of the following statements about f is supported by the data in the table?

(A) The function f is logarithmic because the values of x and the values of $\ln y$ both form arithmetic sequences.

(B) The function f is linear because the values in each column form an arithmetic sequence.

(C) The function f is exponential because the values of x and the values of $\ln y$ both form arithmetic sequences. ✓

(D) The function f is exponential because the values of $\ln y$ increase faster than the values of x.

Answer C

Correct. Both rows form arithmetic sequences and if $y = f(x)$ is exponential, then $\ln y = \ln(f(x))$ is linear.

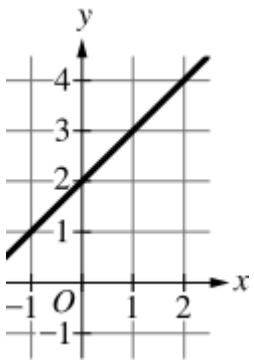
68.

x	0	1	2	3
$\log_3(f(x))$	2	3	4	5

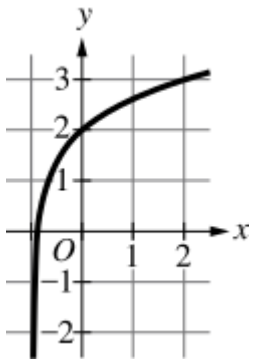
Consider the function f. The table gives values of $\log_3(f(x))$ for selected values of x. Which of the following is a graph of $y = f(x)$?

MCQ_Review_Unit_2

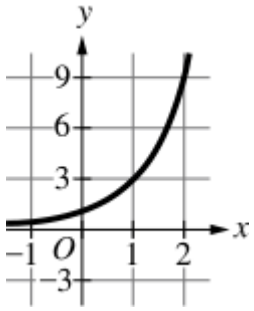
(A)



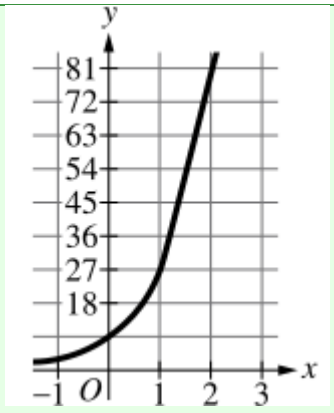
(B)



(C)



(D)

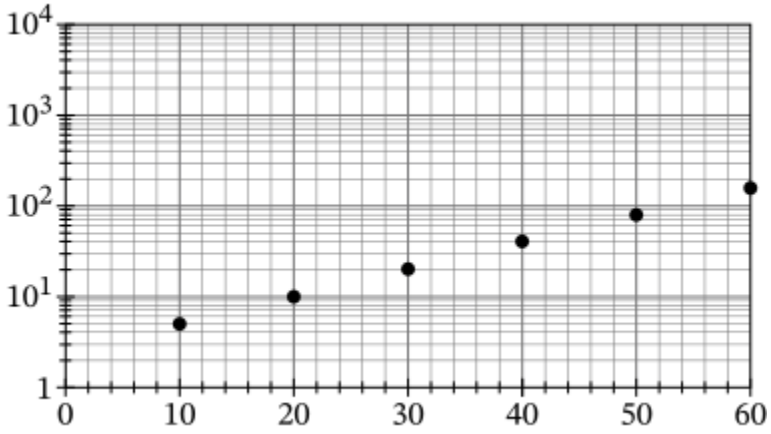


MCQ_Review_Unit_2

Answer D

Correct. This is the result of determining the linear relationship $\log_3(f(x)) = x + 2$ from values in the table. Solving for $f(x)$ gives $f(x) = 9 \cdot 3^x$, an exponential function with base 3 and initial value 9.

69.



The number of thousands of people that have visited a new website is recorded every 10 days for 60 days. These data are used to produce a semi-log plot as shown. The function N gives the number of thousands of people that have visited the website for day t. Which of the following could define $N(t)$?

(A) $\frac{1}{2}t$

(B) $\frac{1}{10}t + 5$

(C) $2.5 \cdot 2^{(t/10)}$ ✓

(D) $3 + 2^{(t/10)}$

Answer C

Correct. This is the result of recognizing that the number of thousands of people doubles every 10 days and using the point $(10,5)$ to determine the initial value.

70. A family needs to buy one shovel and between one and eight plants, inclusive, for their new garden. The cost of the shovel is s dollars, and the cost of one plant is p dollars. The output values of which of the following give the possible costs for these items, in dollars? (Note: Assume any taxes are included in the costs.)

MCQ_Review_Unit_2

(A) The linear function $C(x) = s + px$ for $1 \leq x \leq 8$

(B) The exponential function $C(x) = s \cdot p^x$ for $1 \leq x \leq 8$

(C) The arithmetic sequence $C_n = s + pn$ for $1 \leq n \leq 8$ ✓

(D) The geometric sequence $C_n = s \cdot p^n$ for $1 \leq n \leq 8$

Answer C

Correct. The possible costs fall into a pattern of an initial value and a common difference, making an arithmetic sequence (the domain of which is the positive integers) an appropriate model for the contextual scenario.

71. Which of the following includes the input-output pairs $(2,4)$ and $(3,8)$?

(A) The arithmetic sequence $a_n = 4n$

(B) The linear function $f(n) = 2 + 4(n-1)$

(C) The geometric sequence $g_n = 2^{(n-1)}$

(D) The exponential function $h(n) = 2 \cdot 2^{(n-1)}$ ✓

Answer D

Correct. The output values correspond to the appropriate input values for both of the given input-output pairs.

72. The second term of a sequence is 6, and the fourth term is 24. Of the following, which statement is true?

(A) If the sequence is geometric, the first term could be 1.

(B) If the sequence is arithmetic, the third term could be 12.

(C) If the sequence is geometric, the fifth term could be 48. ✓

(D) If the sequence is arithmetic, the sixth term could be 48.

Answer C

Correct. If the sequence is geometric, the common ratio could be 2, and the fifth term would be 2 times the fourth term. Note that the common ratio could be -2 and satisfy the given conditions; in that case,

the fifth term would be -48.

73. An exponential function G has a known common ratio of $\frac{1}{2}$ and includes the input-output pair $(1,4)$. Which of the following could define $G(x)$?

(A) $4+\frac{1}{2}(x-1)$

(B) $\frac{1}{2} \cdot 4^x$

(C) $4 \cdot \left(\frac{1}{2}\right)^x$

(D) $4 \cdot \left(\frac{1}{2}\right)^{(x-1)}$ ✓

Answer D

Correct. This has the form of an exponential function with the appropriate common ratio, and the initial value index is correctly accounted for in the exponent.

74. The general term of a sequence is given by $a_n = 51 + 3(n-10)$, where a_0 is the initial value. Which of the following expressions also gives the general term of the sequence?

(A) $10 + 3(51-n)$

(B) $17 + 3n$

(C) $21 + 3n$ ✓

(D) $51 \cdot 3^{(n-10)}$

Answer C

Correct. This is the result of rewriting this arithmetic sequence in terms of an initial value and common difference.

t (months)	0	1	2	3	4
$P(t)$ (thousands)	20	30	45	67.5	101.25

The increasing function P gives the number of followers, in thousands, for a new musical group on a social media site. The table gives values of $P(t)$ for selected values of t, in months, since the musical group created their account on this social media site.

75. If a model is constructed to represent these data, which of the following best applies to this situation?

(A) $y = 10t + 20$

(B) $y = \frac{325}{16}t + 20$

(C) $y = 20\left(\frac{2}{3}\right)^t$

(D) $y = 20\left(\frac{3}{2}\right)^t$ ✓

Answer D

Correct. The model that best applies to these data is an exponential growth model $y = ab^x$ with an initial value of 20 and base of $\frac{3}{2}$.