

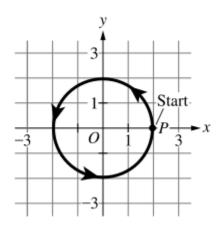
A physical therapy center has a bicycle that patients use for exercise. The height, in inches (in), of the bicycle pedal above level ground periodically increases and decreases when used. The figure gives the position of the pedal P at a height of 12 inches above the ground at time t = 0 seconds. The pedal's 8-inch arm defines the circular motion of the pedal.

- 1. If a patient pedals 1 revolution per second, which of the following could be an expression for h(t), the height, in inches, of the bicycle pedal above level ground at time t seconds?
  - (A)  $8 12 \sin t$ (B)  $12 - 8 \sin t$
  - (C)  $8 12\sin(2\pi t)$

(D)  $12 - 8\sin(2\pi t)$ 

# **Answer D**

Correct. The function model is of the form  $y = a \sin(bt) + d$ . The midline is the position of the pedal at time t = 0 seconds (h(0) = 12), so d = 12. The amplitude of the function is determined by the length of the pedal's arm (8 inches), so |a| = 8. The maximum and minimum values for the height are 12 + 8 and 12 - 8. The period is 1 because the patient pedals 1 revolution per second and  $\frac{2\pi}{b} = 1 \rightarrow b = 2\pi$ . Because of the direction of the motion of the pedal, a = -8.



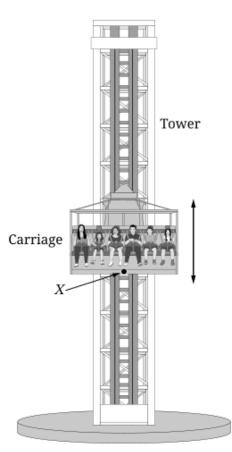
A large wheel of radius 2 feet is rotated at a constant rate. The figure provides a representation of the wheel in the xy-plane with the direction of rotation indicated. At time t = 0 minutes, the wheel begins to rotate. Point P on the wheel is at the "Start" position in the figure. At time t = 20 minutes, 120 rotations of the wheel have been completed, and P is in the same position as it was at time t = 0. A sinusoidal function is used to model the y-coordinate of the position of P as a function of time t in minutes.

- 2. Which of the following functions is an appropriate model for this situation?
  - (A)  $f(t) = 2\sin(\frac{\pi}{10}t)$
  - (B)  $f(t) = 2\sin\left(\frac{\pi}{3}t\right)$
  - (C)  $f(t) = 2\sin(6t)$
  - (D)  $f(t) = 2\sin(12\pi t)$

# **Answer D**

Correct. The function is of the form  $f(t) = a \sin(bt)$ . There are no horizontal or vertical translations. Because 120 rotations complete in 20 minutes, the frequency is 6 rotations per minute. Therefore, the period (the reciprocal of frequency) is  $\frac{1}{6}$  minute per rotation. This gives  $\frac{2\pi}{b} = \frac{1}{6}$  and  $b = 12\pi$ . The amplitude is 2, the radius of the large wheel, so a = 2.

3.



Note: Figure not drawn to scale.

A theme park thrill ride involves a tower and a carriage that rapidly moves passengers up and down along a vertical axis, as shown in the figure. The carriage is lifted to the top of the tower, then released to move down the tower. The ride involves 10 controlled bounces from the highest point to the lowest point, and back to the highest point. A point X is located on the bottom of the carriage. The height of X above the ground, in feet, can be modeled by a periodic function H. At time t = 0 seconds, X is at its highest point of 120 feet. The lowest point for X is at a height of 20 feet. The next time X is at its highest point is at time t = 8 seconds, which is the end of the first bounce. Which of the following can be an expression for H(t), where t is the time in seconds?

(A) 
$$H(t) = 50 \sin(\frac{\pi}{4}t) + 70$$

(B) 
$$H(t) = 50 \cos(\frac{\pi}{4}t) + 70$$

(C) 
$$H(t) = 50 \sin\left(rac{\pi}{8}t
ight) + 70$$

(D) 
$$H(t) = 50 \cos(\frac{\pi}{8}t) + 70$$

# Answer **B**

Correct. The difference between the highest and lowest height for X is 120 - 20 = 100 feet, which

means that the amplitude equals  $\frac{100}{2} = 50$  feet. The midline is y = 70, which means that the vertical shift is 70 feet. X completes one full cycle in 8 seconds, which is the period. Because the value of H(0) is a maximum height of point X, a cosine model appropriately models the scenario, and there is no need for a phase shift. If  $H(t) = a \cos(bt) + d$ , then a = 50,  $\frac{2\pi}{b} = 8$  results in  $b = \frac{\pi}{4}$ , and d = 70. Therefore,  $H(t) = 50 \cos(\frac{\pi}{4}t) + 70$ .

- 4. If The rational function r is given by  $r(x) = \frac{x^3 4x + 3}{x^4 + 2x 4}$ . For what values of x does r(x) = 0?
  - (A) x = -2.303 and x = 1.000 only
  - (B) x = -1.643 and x = 1.144 only
  - (C) x = -2.303, x = 1.000, and x = 1.303 only
  - (D) x = -2.303, x = -1.643, x = 1.000, x = 1.303, and x = 1.144

### Answer C

Correct. The graphing calculator is used to find the x-intercepts of the rational function (when the numerator is zero and the denominator is not zero).

The function f is given by  $f(x) = \sin(2.25x + 0.2)$ . The function g is given by g(x) = f(x) + 0.5.

- 5. If What are the zeros of g on the interval  $0 \le x \le \pi$ ?
  - (A) 1.085 and 2.481
  - (B) 1.307 and 2.704
  - (C) 1.540 and 2.471
  - (D) 0.144, 1.075, and 2.936

# Answer C

Correct. Applying the transformation to function g results in  $g(x) = \sin(2.25x + 0.2) + 0.5$ . This is a vertical translation of the graph of f 0.5 units up. Using a graphing calculator, the zeros are found by solving for g(x) = 0, finding the x-intercepts of the graph of g.

- 6. If The function f is given by  $f(x) = 4 \cdot 3^{(x-2)} + 1$ . The function g is given by  $g(x) = f^{-1}(x)$ . For which of the following values does g(x) = -3x?
  - (A) -0.426
  - (B) 1.016
  - (C) 1.025
  - (D) 1.444

### Answer B

Correct. Finding the inverse function of f results in  $g(x) = \log_3(\frac{x-1}{4}) + 2$ . Using the graphing calculator, the point of intersection of the graphs of y = g(x) and y = -3x is (1.016, -3.047). Therefore, x = -1.016. Alternately, because f and g are inverses, this value is also the y-coordinate of the point of intersection of the graphs y = f(x) and  $y = -\frac{1}{3}x$ , the inverse function y = -3x. These graphs intersect at (-3.047, 1.016).

### 7.

Day	0	5
Amount (grams)	2	6.315

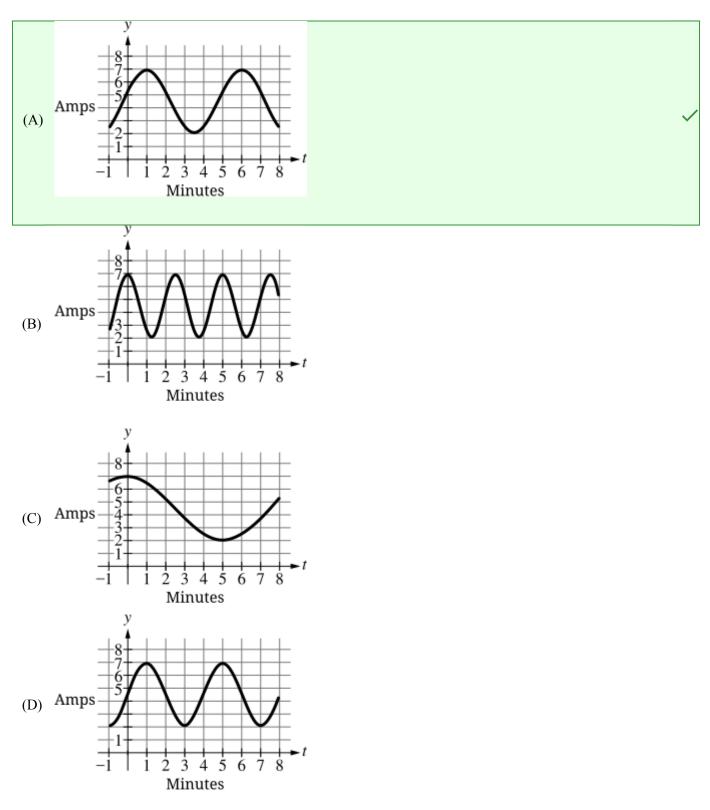
The table gives values for the amount of a certain substance, in grams, on certain days. The data are modeled by an exponential function f given by  $f(t) = 2e^{(r \cdot t)}$ , where t is the number of days since day 0. The constant r is defined as the continuous growth rate of this model. Based on the table, what is the value of r?

- (A) 0.230
- (B) 0.259
- (C) 0.863
- (D) 1.259

# Answer A

Correct. Using (5,6.315) from the table results in  $6.315 = 2e^{(r \cdot 5)}$ . The graphing calculator can be used to solve this equation for r.

8. A certain type of machine produces a number of amps of electricity that follows a cyclic, periodically increasing and decreasing pattern. The machine produces a maximum of 7 amps at certain times and a minimum of 2 amps at other times. It takes about 5 minutes for one cycle from 7 amps to the next 7 amps to occur. Which of the following graphs models amps as a function of time, in minutes, for this machine?



# Answer A

Correct. This graph represents the behavior of the function as described, such that one cycle (period) of the function, the distance between two maximum values, is 5 minutes. There are maximum values of 7 amps and minimum values of 2 amps.

9. The fifth term of a geometric sequence is 24, and the sixth term is 48. What is the value of the tenth term?

- (A) 144
- (B) 168

( )		
(C)	768	~
(D)	1536	

# Answer C

Correct. The common ratio of this geometric sequence is 2. From the sixth term, the common ratio needs to be applied to 48 four more times, or by using the fifth term,  $g_{10} = 24 \cdot 2^{(10-5)}$ .

**10.** The first term of an arithmetic sequence is 5, and the common difference of the sequence is 2. What is the eighth term of the sequence?

(A)	19	~
(B)	21	
(C)	640	

(D) 1280

# Answer A

Correct. This is the result of identifying the terms of the arithmetic sequence as  $a_n = 5 + 2(n-1)$ , where  $a_1 = 5$ , and finding  $a_8$ .

11. The terms of the increasing arithmetic sequence  $a_n$  are positive. The terms of the increasing geometric sequence  $g_n$  are positive. The values of the first terms of both sequences are the same, and the values of the fourth terms of both sequences are the same. Which of the following statements describes the values of the second terms of the sequences?

- (A) The second term of the arithmetic sequence must be less than the second term of the geometric sequence.
- (B) The second term of the arithmetic sequence must be greater than the second term of the geometric sequence.
- (C) The second term of the arithmetic sequence must be the same value as the second term of the geometric sequence.
- (D) The relationship between the values of the second terms cannot be determined from the given information.

#### Answer **B**

Correct. Because the terms of both the arithmetic and geometric sequences are positive and increasing, the terms of the arithmetic sequence increase at a constant rate, while the terms of the geometric sequence increase at an increasing rate. Therefore, for the first and fourth terms to have equal values, the geometric sequence values between these two points must be less than the arithmetic sequence values between these two points.

12. If The function S is given by  $S(t) = \frac{500,000}{1+0.4e^{kt}}$ , where k is a constant. If S(4) = 300,000, what is the value of S(12)?

(A)	175,325	~
(B)	214,772	
(C)	343,764	
(D)	357.143	

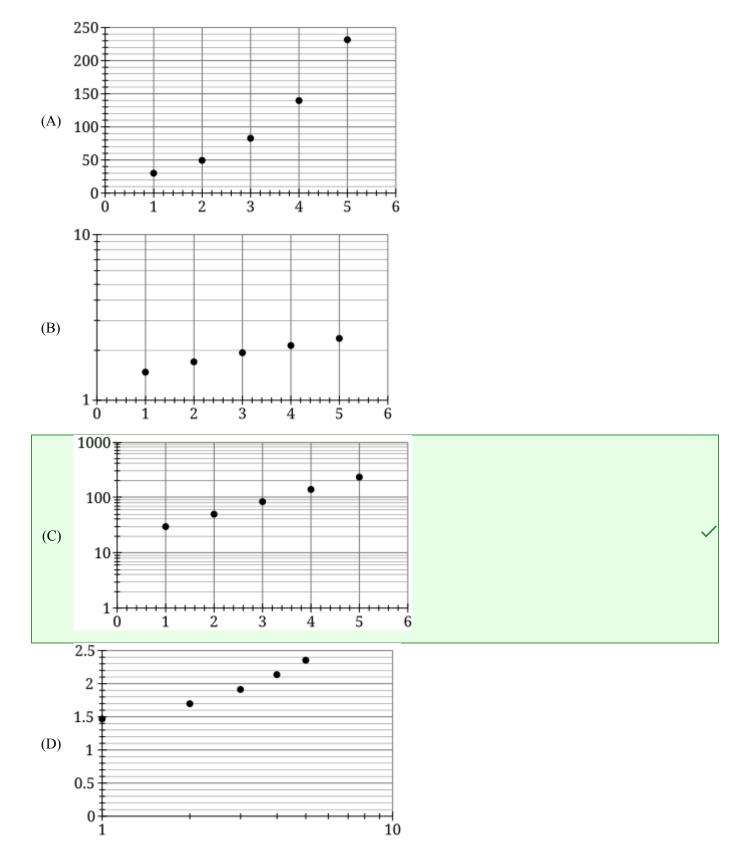
## Answer A

Correct. The value of the constant k is found by using the graphing calculator to solve the equation  $300,000 = \frac{500,000}{1+0.4e^{4k}}$  by finding the point of intersection of the graphs of y = 300,000 and  $y = \frac{500,000}{1+0.4e^{4x}}$ . This results in k = 0.128. Store the full value of k in the graphing calculator and evaluate  $S(12) = \frac{500,000}{1+0.4e^{12k}}$ .

t	1	2	3	4	5
m(t)	30	50	83	139	231

The table gives values for a function m at selected values of t.

13. Which of the following graphs could represent these data in a semi-log plot, where the vertical axis is logarithmically scaled?



# Answer C

Correct. This graph is a plot of the data points (t, m(t)) with the vertical axis using a base 10 logarithmic scale. The data in the table models an exponential function  $y = ab^t$ . Therefore, the data appears linear in a semi-log plot.

# 14.

Time $t$ (hours past 12 midnight)	Temperature (°F)
0	91
6	83
11	92
18	102
23	92

The table gives the temperature, in degrees Fahrenheit, in a certain town on a given day at selected times t, in hours past 12 midnight. A sinusoidal regression model  $y = a \sin(bt + c) + d$  is calculated using all five data points, where time t is the input value and temperature y is the output value. Based on the regression model, which of the following is the best choice for the amplitude of the sinusoidal function?

(A) 9.5

(B)	9.8	~
(C)	19.5	
(D)	92.5	

#### Scoring Guide

# MCQ\_Review\_Calc

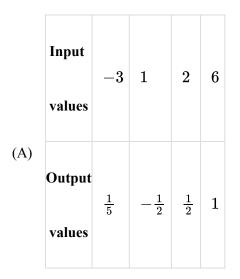
# Answer B

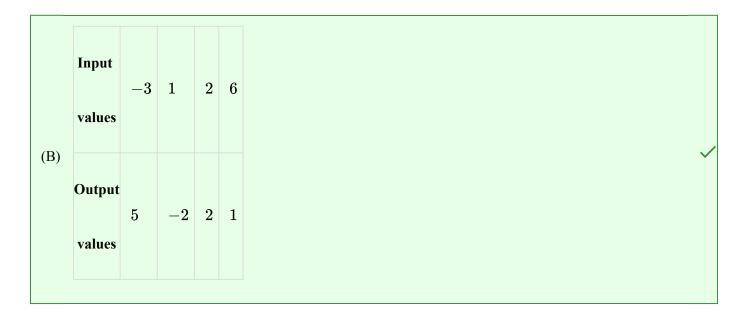
Correct. This is the value of a in the sinusoidal regression.

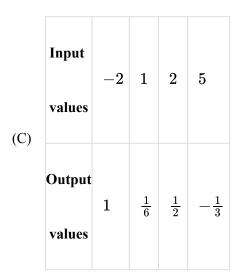
15.

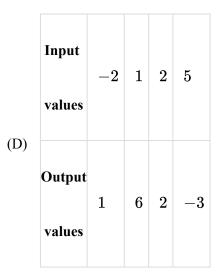
$x \ -2$	1	2	5
y  1	6	2	-3

The table gives values for the invertible function y = f(x) for selected values of x. Which of the following inputoutput pairs describes  $f^{-1}$ ?









# **Answer B**

Correct. These input-output pairs reverse the mapping of f.

# 16. 🔢

Year	2002	2004	2005	2008	2011	2013	2014
Surcharge (dollars)	1.38	1.37	1.54	1.97	2.40	2.60	2.77

The table gives the average automated teller machine (ATM) surcharge fee, in dollars, in the United States for selected years from 2002 to 2014. An exponential regression  $y = ab^t$  is used to model these data, where t = 2 corresponds to 2002. Based on the exponential model, what is the error in the model, in dollars, for 2013, and is the value predicted by the model for 2013 an underestimate or overestimate of the surcharge fee?

- (A) The error is 0.01, and the value predicted by the model is an underestimate.
- (B) The error is 0.01, and the value predicted by the model is an overestimate.
- (C) The error is 0.03, and the value predicted by the model is an underestimate.
- (D) The error is 0.03, and the value predicted by the model is an overestimate.

# Answer D

Correct. The value predicted by the model is about 2.63. The error is the predicted value minus the actual value from the table, 2.60. The predicted value overestimates the actual value by 0.03.

- 17. The speed of a car traveling on a highway is being recorded once per second for two minutes. During this time interval, the car gradually speeds up slightly to pass another vehicle, then the car returns to its original speed. The recorded speed of the car with respect to time can be modeled by linear, quadratic, and exponential functions. For each of the three models, their residuals are small and are without pattern. Which of the following conclusions is best?
  - (A) A linear model is best based on contextual clues.
  - (B) A quadratic model is best based on contextual clues.
  - (C) An exponential model is best based on contextual clues.
  - (D) Contextual clues fail to help in selecting a model for this contextual situation.

# **Answer B**

Correct. Because the car speeds up and slows down, a model with the potential for symmetry and a maximum value for the two minutes is best.

18.

III Year	US Federal Education Spending (billions of dollars)
2011	\$112.8
2012	\$109.3
2013	\$105.1
2014	\$104.5
2015	\$99.0
2016	\$99.3
2017	\$97.7

The table gives amounts of United States federal education spending, in billions of dollars, for selected years. A linear regression is used to construct a function model S that models the spending, in billions of dollars, over the given years. If t = 1 corresponds to 2011, t = 2 corresponds to 2012, and this pattern continues, which of the following defines function S?

(A)	S(t) =	-2.55t + 3	114.157
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- (B) S(t) = -2.55t + 5239.657
- (C) S(t) = -8.099t + 113.820
- (D) S(t) = -0.369t + 42.308

# Answer A

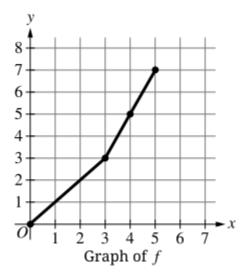
Correct. This is the result of calculating the linear regression model for the table of values using a graphing calculator. The input variable t represents the number of years after 2010.

- 19. If the function f is given by  $f(x) = x^2 + 3x 5$ . Which of the following describes f?
  - (A) For any interval of x, the function always has a positive rate of change.
  - (B) For any interval of x, the function always has a negative rate of change.
  - (C) For any interval of x < -1.5, the function has a positive rate of change, and for any interval of x > -1.5, the function has a negative rate of change.

(D) For any interval of x < -1.5, the function has a negative rate of change, and for any interval of x > -1.5, the function has a positive rate of change.

# Answer D

Correct. The graph of the function opens up. The x-value of the vertex of the quadratic function is x = -1.5. As x increases for x < -1.5, the outputs decrease (a negative rate of change) and as x increases for x > -1.5, the outputs increase (a positive rate of change).



The graph of the piecewise-linear function f is shown in the figure. Let g be the inverse function of f.

### **20.** What is the maximum value of g?

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# MCQ\_Review\_Calc

(A) 
$$\frac{1}{7}$$

 (B)  $\frac{1}{5}$ 

 (C) 5

 (D) 7

# Answer C

Correct. By reversing the roles of x and y of function f, the values of the inverse function g can be found. The value f(5) = 7 results in g(7) = 5, the maximum value of function g. The graph of g is a reflection of the graph of f over the identity function y = x.

x	-2	-1	0	1	2
f(x)	1	2	-1	-2	0
g(x)	2	0	1	-1	0

The table gives values for the functions f and g at selected values of x. Functions f and g are defined for all real numbers. Let h be the function defined by h(x) = f(g(x)).

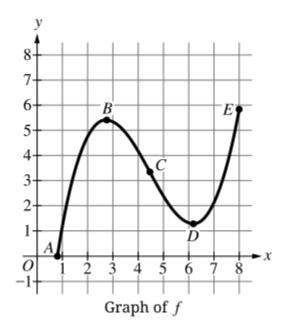
#### **21.** What is the value of h(0)?

(A) -2		$\checkmark$
(B) -1		
(C) 0		

(D) 2

# Answer A

Correct. The value of h(0) is h(0) = f(g(0)) = f(1) = -2.



The figure shows the graph of a function f. The zero and extrema for f are labeled, and the point of inflection of the graph of f is labeled. Let A, B, C, D, and E represent the x-coordinates at those points.

22. Of the following, on which interval is f increasing and the graph of f concave down?

$(\Lambda)$	the interval from $A$ to $B$	
(A)	the interval from $A$ to $D$	

- (B) the interval from B to C
- (C) the interval from C to D
- (D) the interval from D to E

# Answer A

Correct. The function is increasing, and the graph of the function is concave down on this interval.

23. If A ball is thrown through an open window to the ground below. The height of the ball, in meters, at time t seconds after it is thrown can be modeled by the function h, given by  $h(t) = -4.9t^2 + 4.4t + 15.24$ . Which of the following describes the height of the ball above the ground?

- (A) The ball begins at its maximum height of 15.240 meters. The height of the ball decreases until it reaches the ground 1.820 seconds after it leaves the window.
- (B) The ball begins at its maximum height of 15.240 meters. The height of the ball decreases until it reaches the ground 2.269 seconds after it leaves the window.
- After leaving the window, the height of the ball increases to its maximum height of 16.228 meters.
   (C) Then the height of the ball decreases until it reaches the ground 1.820 seconds after reaching its maximum height.

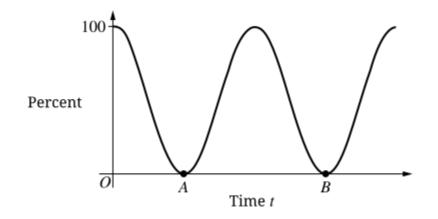
After leaving the window, the height of the ball increases to its maximum height of 16.228 meters.

(D) Then the height of the ball decreases until it reaches the ground 2.269 seconds after reaching its maximum height.

# Answer C

Correct. The graphing calculator is used to find the maximum (0.449, 16.228) and the contextappropriate zero (2.269, 0) of the quadratic function h. The difference between the x-values of the maximum and context-appropriate zero, 1.820 seconds, accurately communicates the time it takes for the ball to hit the ground after reaching its maximum height.

24.



New Moon	Ioon First Quarter Full Moon		Third Quarter	
May 4			May 26	
June 2			June 25	
July 1	July 8	July 16	July 24	

When seen from Earth, the percent of the Moon illuminated by the Sun varies. At the full moon phase, 100% of the Moon is illuminated, while at the new moon phase 0% of the Moon is illuminated. The graph gives the percent of the Moon illuminated at time t, in days, since an initial day, along with two labeled points A and B. The table gives the dates of the four consecutive, periodic moon phases for three months of a certain year. Approximately how many days occur between points A and B? (Note: Assume that a year has 365 days and consists of 12 months.)

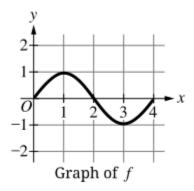
- (A) 7
- (B) 14

(C)	) 28	~
(D)	) 56	

# Answer C

Correct. The approximate number of days from new moon to new moon (point A to point B) is 28 days. Therefore, the period is 28 days.

25.



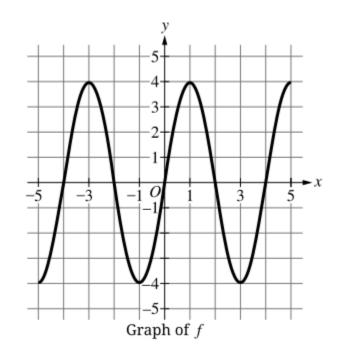
The graph gives one cycle of a periodic function f in the xy-plane. Which of the following describes the behavior of f on the interval 39 < x < 41?

- (A) The function f is decreasing.
- (B) The function f is increasing.
- (C) The function f is decreasing, then increasing.
- (D) The function f is increasing, then decreasing.

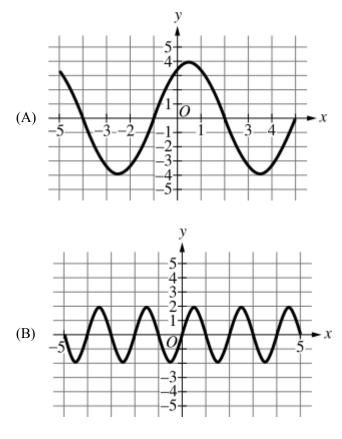
# **Answer B**

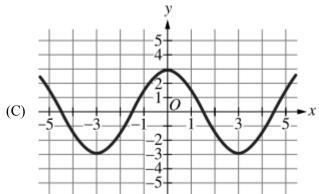
Correct. Because f is periodic and the length of a cycle is 4, the graph of f on 39 < x < 41 has the same behavior as the graph of f on 3 < x < 5. This is the same behavior on 3 < x < 4, followed by 0 < x < 1. On both of these intervals, the function is increasing.

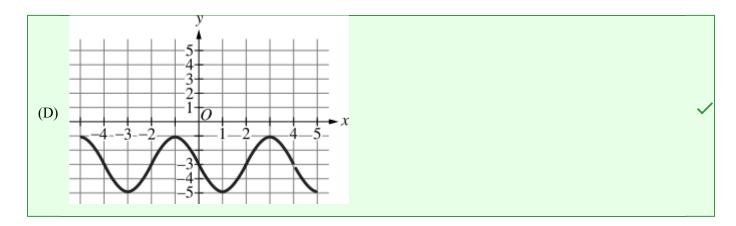
26.



The graph of the function f is given in the xy-plane. Which of the following functions has the same period as f?







### Answer D

Correct. The period of both functions is 4.

27.

Pressure (P)	137.500	103.125	82.500	68.750	58.929	51.563	45.833	41.250
Volume (V)	12	16	20	24	28	32	36	40

# ...

Boyle's Law states that the pressure of a gas is inversely proportional to the volume of the gas at a constant temperature. The table gives the volume V, in milliliters (mL), of a gas for selected pressures P. Which of the following gives a model for the volume of the gas as a function of pressure? (Note: the units for pressure are not included.)

- (A) V(P) = 11.458P
- (B) V(P) = 1650P
- (C)  $V(P) = \frac{11.458}{P}$

(D)  $V(P) = \frac{1650}{P}$ 

# Answer D

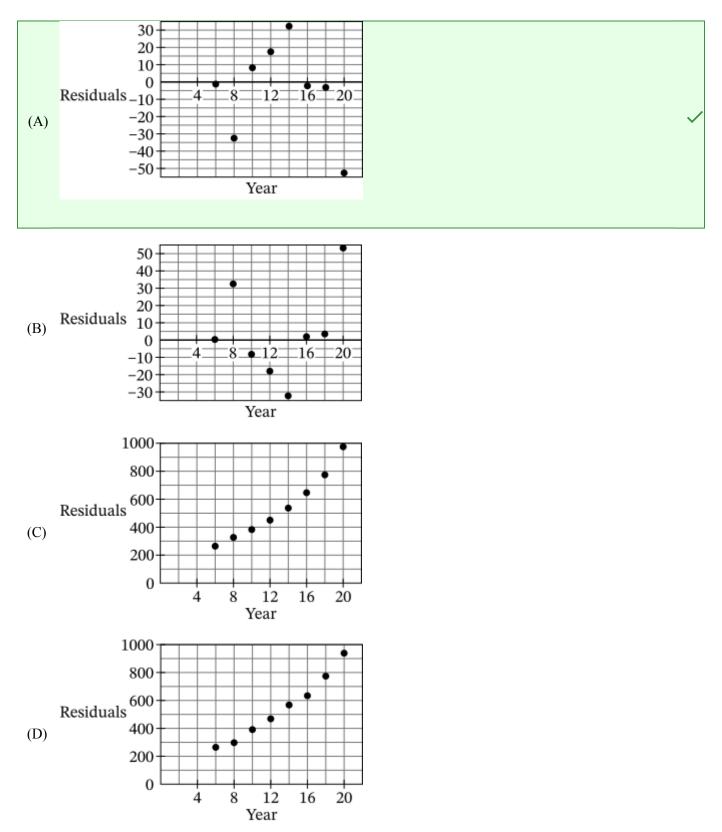
Correct. The rational function model,  $V(P) = \frac{k}{P}$ , is determined by using the constant of proportionality k = 1650 for the given data, where  $k = P \cdot V$ .

28.

....

Year	Number of Vehicles (thousands)
1996	265.0
1998	295.0
2000	394.7
2002	471.1
2004	565.5
2006	634.6
2008	775.7
2010	938.6

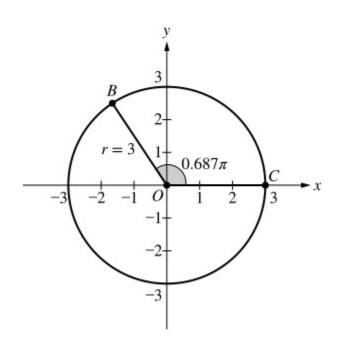
The table gives the number of alternative-fuel vehicles, in thousands, in use in the United States for selected years from 1996 to 2010. The function A defined by  $A(t) = \frac{4615t - 8727}{-0.542t^2 + 15t + 1}$  is used to model the data, in thousands, for  $6 \le t \le 20$ , where t = 6 corresponds to 1996. Which of the following could be a plot of the residuals of the model?



# Answer A

Correct. This is the result of calculating the residuals (the actual values minus the predicted values) of the model and creating the corresponding plot of residuals. Using the graphing calculator, store the values from the table in one list. Store the values of A(t) using the same t-values from the table in another list. In a third list, calculate the differences between the list entries, actual values (from the table) minus predicted values (from A(t)).

29.



The figure gives an angle in standard position with measure  $0.687\pi$  and a circle with radius 3 in the xy-plane. What is the length of the minor arc of the circle from point C to point B, the arc subtended by the angle?

- (A) 0.719
- (B) 2.158

(C)	6.475			

(D) 9.712

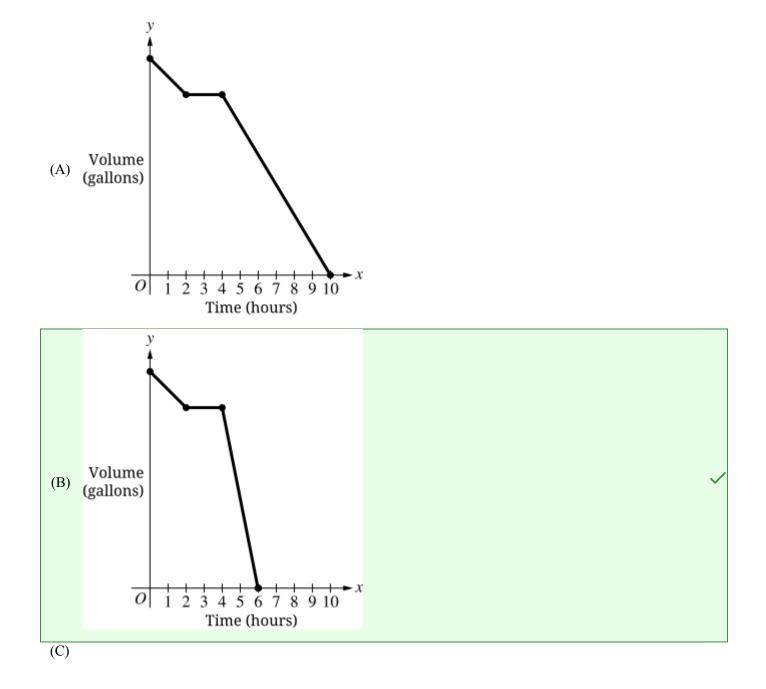
# Answer C

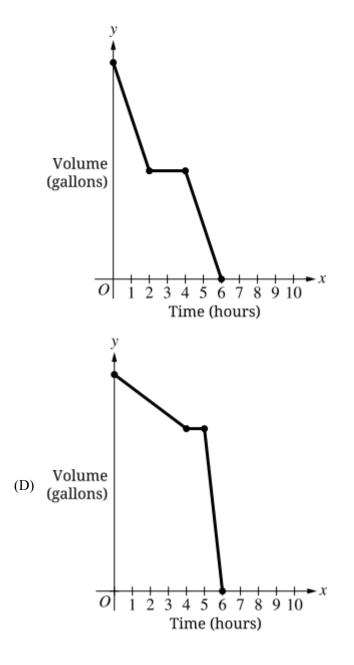
Correct. The angle is the ratio of the arc to the radius, making the length of the arc the product of the angle and the radius,  $0.687\pi \cdot 3$ .



The figure shows a swimming pool filled with water. A pump is used to remove water from the pool until the pool is empty. When the pump is running, the rate at which the volume of water in the pool decreases is constant. During the first two hours, the pump works slower than usual due to a broken piece. Then the pump stops working. The broken piece is replaced, and the pump works at its usual rate until the pool is completely emptied of water. The entire process of emptying the pool takes six hours.

**30.** Which of the following graphs could depict this situation, where time, in hours, is the independent variable, and the volume of water in the pool, in gallons, is the dependent variable?





# Answer B

Correct. This graph accurately depicts the verbal description of the emptying of the pool, taking into account the varying rates at which it emptied and the amount of time it emptied at each rate.

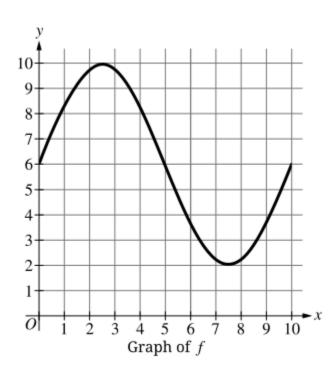
- 31. The Richter scale is a numerical scale that uses base 10 logarithms for measuring an earthquake's magnitude. The larger the number, the more intense the earthquake. For example, an earthquake with a magnitude of 5.0 is 10 times more intense than an earthquake with a magnitude of 4.0. Two well-known earthquakes occurred in the year 1906. The San Francisco earthquake had a magnitude of 7.9, and the Chile earthquake had a magnitude of 8.2. Approximately how many times more intense was the Chile earthquake than the San Francisco earthquake?
  - (A) 0.3
  - (B) 0.5

(C) 2.0	$\checkmark$
(D) <b>3.0</b>	

### Answer C

Correct. The difference in the magnitudes is 0.3, so the increase in intensity on a base 10 logarithmic scale is  $10^{0.3} \approx 1.995$ .

32.



The graph of the sinusoidal function f is given in the xy-plane. Of the following, which is greatest?

- (A) The length of the interval that satisfies  $f(x) \le 5$
- (B) The sum of the lengths of the intervals that satisfy  $5 \le f(x) \le 6$
- (C) The length of the interval that satisfies  $f(x) \ge 9$
- (D) The length of the interval that satisfies  $f(x) \ge 10$

# Answer A

Correct. By inspection of the graph, this is an interval of length that is approximately 4.

- **33.** Consecutive terms of a sequence have the values 6, 2, -2, and -6. Of the following, which describes the sequence?
  - (A) The terms could be part of an arithmetic sequence with a common difference of -4.
  - (B) The terms could be part of a geometric sequence with a common difference of -4.
  - (C) The terms could be part of an arithmetic sequence with a common ratio of -4.
  - (D) The terms could be part of a geometric sequence with a common ratio of -4.

#### Answer A

Correct. An arithmetic sequence consists of successive terms that have a common difference, or constant rate of change. These particular consecutive terms have a common difference of -4.

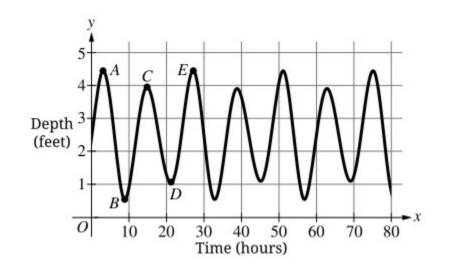
34. The daily high temperatures of a certain city over a period of time are modeled with a sinusoidal function in the xy -plane. The maximum daily high temperature is 80°F, and the minimum daily high temperature is 55°F. Based on these temperatures, which of the following is the best value for the amplitude of the sinusoidal function?

	12.5	~
(B)	25	
(C)	40	
(D)	67.5	

# Answer A

Correct. The amplitude is half the difference between the maximum and minimum values.  $\frac{80-55}{2} = \frac{25}{2}$ 

35.



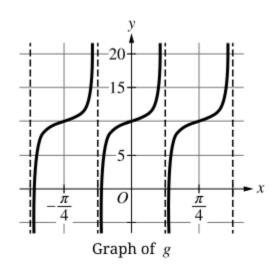
A graph of the depth of water at a pier in the ocean is given, along with five labeled points A, B, C, D, and E in the xy-plane. For the time periods near these data points, a periodic relationship between depth of water, in feet, and time, in hours, can be modeled using one cycle of the periodic relationship. Based on the graph, which of the following is true?

- (A) The time interval between points A and B gives the period.
- (B) The time interval between points A and C gives the period.
- (C) The time interval between points A and D gives the period.
- (D) The time interval between points A and E gives the period.

### Answer D

Correct. One cycle of the periodic relationship starts at point A and begins repeating at point E.

36.



The graph of the function g is given in the xy-plane. If g is the result of multiplicative and additive transformations of the graph of  $y = \tan x$ , what is the period of g?

(A) $\frac{\pi}{8}$	
(B) $\frac{\pi}{4}$	$\checkmark$
(C) $\frac{\pi}{2}$	
(D) $\pi$	

### Answer B

Correct. The graph of function g repeats every  $\frac{\pi}{4}$  units, making this the value of the period.

**37.** A metronome is a practice tool that produces a steady beat to help musicians play rhythms accurately. Some metronomes use a pendulum that repeatedly swings left and right to produce the steady beat. Which of the following is true about the beat produced by the metronome and the motion of the pendulum?

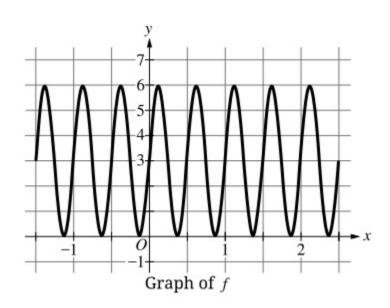
(A) Both the beat produced by the metronome and the motion of the pendulum have periodic relationships with time.

- (B) Only the beat produced by the metronome has a periodic relationship with time.
- (C) Only the motion of the pendulum has a periodic relationship with time.
- (D) Neither the beat produced by the metronome nor the motion of the pendulum have periodic relationships with time.

## Answer A

Correct. As time increases, both the beat and the motion demonstrate a repeating pattern over successive time intervals.

38.



The figure shows the graph of a periodic function f in the xy-plane. What is the frequency of f?

(A) 0.5

(B)	2	$\checkmark$
(C)	3	

(D) 8

## Answer B

Correct. Based on points on the graph such as (0,3) and (0.5,3), the period of the sinusoidal function is 0.5. The frequency is the reciprocal of the period, so the frequency is  $\frac{1}{0.5} = 2$ . Alternately, based on the points (0,3) and (1,3), the sinusoidal function completes 2 cycles per 1 unit of the *x*-axis.

**39.** Consecutive terms of a sequence have the values 2, -1,  $\frac{1}{2}$ ,  $-\frac{1}{4}$ , and  $\frac{1}{8}$ . Of the following, which describes the sequence?

- (A) The terms could be part of a geometric sequence with a common ratio of -2.
- (B) The terms could be part of a geometric sequence with a common ratio of  $-\frac{1}{2}$ .
- (C) The terms could be part of a geometric sequence with a common ratio of  $\frac{1}{2}$ .
- (D) The terms are part of a sequence other than a geometric sequence because the common ratio of a geometric sequence must be positive.

## **Answer B**

Correct. The terms demonstrate a common ratio (or a constant proportional change) of -1 \_ (1/2) \_ (-1/4) \_ (1/8) \_ 1

$$\frac{1}{2} = \frac{1}{-1} = \frac{1}{(1/2)} = \frac{1}{(-1/4)} = -\frac{1}{2}$$

- 40. The number of hours of daylight per day in a certain town can be modeled by a sinusoidal function that is graphed in the xy-plane. Over the course of a year, the greatest number of hours of daylight per day is 14.5 hours, and the least number of hours of daylight per day is 7 hours. The line y = d, where d is a constant, is the midline of the graph of the sinusoidal function. Which of the following is the best value for d? (Note: Assume that 1 year is 365 days.)
  - (A) **3.75**
  - (B) 7.5
    (C) 10.75
    (D) 365

## Answer C

Correct. This is the arithmetic mean (average) of the maximum and minimum values. This average determines the value of the midline of the graph.

#### 41.

θ	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$
r=f( heta)	0	$\frac{5}{2}$	$\frac{5\sqrt{3}}{2}$

The table gives values of a polar function  $r = f(\theta)$  for selected values of  $\theta$ . If the value of  $r = f(\frac{\pi}{12})$  is estimated using the average rate of change of the function over the interval  $0 \le \theta \le \frac{\pi}{6}$ , which of the following is true?

(A) The estimated value would be an overestimate of the actual value by approximately 0.223.

(B) The estimated value would be an underestimate of the actual value by approximately 0.223.

(C) The estimated value would be an overestimate of the actual value by approximately 0.335.

(D) The estimated value would be an underestimate of the actual value by approximately 0.335.

#### Answer D

Correct. The average rate of change of the function over the interval  $0 \le \theta \le \frac{\pi}{6}$  is

 $\frac{\left(5\sqrt{3}/2\right)-0}{(\pi/6)-0} = \frac{5\sqrt{3}}{2} \cdot \frac{6}{\pi} = \frac{15\sqrt{3}}{\pi}.$  The estimated value of  $r = f\left(\frac{\pi}{12}\right)$  found using this average rate of change is  $r = f\left(\frac{\pi}{12}\right) \approx 0 + \frac{15\sqrt{3}}{\pi}\left(\frac{\pi}{12} - 0\right) = \frac{15\sqrt{3}}{12} \approx 2.165.$  This value is about 0.335 less than  $\frac{5}{2} = 2.5.$ 

42. If At a coastal city, the height of the tide, in feet (ft), is modeled by the function h, defined by  $h(t) = 6.3 \cos(\frac{\pi}{6}t) + 7.5$  for  $0 \le t \le 12$  hours. Based on the model, which of the following is true?

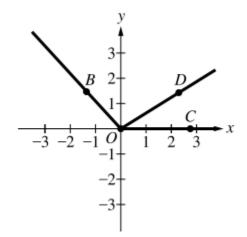
(A) The maximum height of the tide is 13.8 ft.

- (B) The maximum height of the tide occurs at t = 6 hours.
- (C) The minimum height of the tide is 1 ft.
- (D) The minimum height of the tide occurs at t = 12 hours.

#### Answer A

Correct. The maximum height of the tide can be found by graphing function h, and finding the maximum of 13.8 ft occurs at t = 0 and at t = 12. The maximum value is equal to the value of the midline, y = 7.5, plus the value of the amplitude 6.3. One can also reason that the maximum value of cosine is 1, so the maximum value of h occurs when cosine is 1. Therefore, the maximum value is  $6.3 \cdot 1 + 7.5$ .

43.



The figure gives four points and some corresponding rays in the xy-plane. Which of the following is true?

- (A) Angle COB is in standard position with initial ray OB and terminal ray OC.
- (B) Angle COB is in standard position with initial ray OC and terminal ray OB.
- (C) Angle DOB is in standard position with initial ray OB and terminal ray OD.
- (D) Angle DOB is in standard position with initial ray OD and terminal ray OB.

#### Answer B

Correct. The vertex of angle COB corresponds with the origin, the initial ray is OC because it coincides with the positive x-axis, and the terminal ray is OB.

44. 🏢

Month (x)	Debt (y) (dollars)
1	620
4	1,083
5	$1,\!215$
7	$1,\!902$

The table gives the amount of debt, in dollars, on an individual's credit card for certain months after opening the credit card. Using an exponential regression  $y = ab^x$  to model these data, what is the debt at month 24 predicted by the exponential function model, to the nearest dollar? (Assume that the debt continues and that no payments are made to reduce the debt.)

- (A) 5,267
- (B) 15,187
- (C) 42,159
- (D) 1,972,745

## Answer C

Correct. An exponential regression is used and the output value for an input value of 24 is calculated. The regression model is  $y = 510.866(1.20187)^x$ 

45. If The temperature, in degrees Celsius (°C), in a city on a particular day is modeled by the function T defined by  $T(t) = \frac{75t^3 - 836t^2 + 3100t - 4185}{14t^2 + 10t - 35}$ , where t is measured in hours from 12 p.m. for  $2 \le t \le 9$ . Based on the model, how many hours did it take for the temperature to increase from 0°C to 5°C?

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- (A) 7.701
- (B) 5.420
- (C) 4.114
- (D) 2.280

## Answer D

Correct. To answer the question, find the difference in the *t*-values when the temperature is 0 degrees and when the temperature is 5 degrees. For the interval  $2 \le t \le 9$ , use the graphing calculator to find the zero of T and the point of intersection of the graph of T and the horizontal line t = 5. The zero is at t = 5.4200, and the point of intersection is at (7.7007, 5). Therefore, 7.7007 - 5.4200 = 2.280 hours.

- 46. The amount of a certain medicine, in milligrams, in a patient's body t hours after an initial dose can be modeled by the exponential decay function f given by  $f(t) = ab^t$ . An initial dose of the medicine is 10 milligrams, and after 2 hours the amount in the body is 5 milligrams. At what time t will the amount in the body be 0.01 milligram? (Assume no additional doses of the medicine are given after the initial dose.)
  - (A) 3.996
  - (B) 4.983
  - (C) 9.966
  - (D) 19.932

## Answer D

Correct. The decay factor of  $\frac{1}{2}$  applied every 2 hours results in the model  $f(t) = 10(\frac{1}{2})^{(t/2)}$ . The equation f(t) = 0.01 is solved with the graphing calculator to give t = 19.931569.

## 47. 🔳

Time $t$ (hours)	1	2	3
Mold (square micrometers)	0.3	0.75	1.875

The table gives the amount of mold, in square micrometers, on a certain food at time t hours. If the amount of mold grows exponentially, what would the amount of mold be, in square micrometers, at time t = 3.75?

(A) 1.538

(B)	3.728	
(C)	4.688	

(D) 9.320

## Answer B

Correct. This is the result of producing an exponential growth model, using a growth factor of  $\frac{0.75}{0.3}$ , using (1,0.3) from the table, and adjusting the exponent for the index to write  $0.3\left(\frac{0.75}{0.3}\right)^{(3.75-1)}$ .

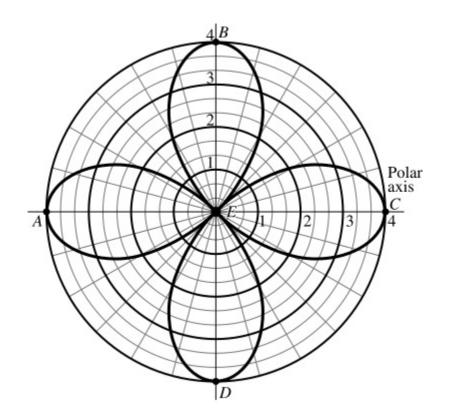
Two function models k and m are constructed to represent the sales of a product at a group of grocery stores. Both k(t) and m(t) represent the sales of the product, in thousands of units, after t weeks for  $t \ge 2$ .

**48.** If  $k(t) = 14 - 2.885 \ln t$  and m(t) = -t + 14, what is the first time t that sales predicted by the logarithmic model will be 0.1 thousand units more than sales predicted by the linear model?

(A) $t = 6.318$	
(B) $t = 4.324$	$\checkmark$
(C) $t = 3.577$	
(D) $t = 2.289$	

## Answer **B**

Correct. One method for solving this question involves defining a new function y = k(t) - m(t), and finding the point of intersection of the graph of this function and the graph of y = 0.1. For  $t \ge 2$ , the graphs intersect at the point where t = 4.324. The other point of intersection is at a value where t < 2.



The figure shows the graph of the polar function  $r = f(\theta)$ , where  $f(\theta) = 4\cos(2\theta)$ , in the polar coordinate system for  $0 \le \theta \le 2\pi$ . There are five points labeled A, B, C, D, and E. If the domain of f is restricted to  $0 \le \theta \le \frac{\pi}{2}$ , the portion of the given graph that remains consists of two pieces. One of those pieces is the portion of the graph in Quadrant I from C to E.

- 49. Which of the following describes the other remaining piece?
  - (A) The portion of the graph in Quadrant I from E to B
  - (B) The portion of the graph in Quadrant II from E to A
  - (C) The portion of the graph in Quadrant III from E to A
  - (D) The portion of the graph in Quadrant III from E to D

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## Answer D

Correct. This piece of the graph is on the domain of f with  $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$ .

In a certain simulation, the population of a bacteria colony can be modeled using a geometric sequence, where the first day of the simulation is day 1. The population on day 4 was 4,000 bacteria, and the population on day 8 was 49,000 bacteria.

50. If What was the population of the colony on day 6 based on the simulation?

- (A) **26,500**
- (B) 26,192
- (C) 14,000
- (D) 611

## Answer C

Correct. Using the graphing calculator, a common ratio for the geometric sequence can be found by solving  $4,000r^4 = 49,000$  for r, which results in r = 1.871. Store the full value of r in the graphing calculator to use. On day 6 the population of the colony is  $4,000r^2 = 14,000$  bacteria.

51.

Age of Tree (years)	Height of Tree (feet)
1	4.0
3	7.8
5	9.6
7	10.8
9	11.6
11	12.3
13	12.9

The table gives the height of a tree, in feet, for selected ages of the tree, in years. A logarithmic regression is used to produce a model of the form  $f(x) = a + b \ln x$ , where f(x) gives the predicted height of a tree, in feet, at age x years. Based on the model, what is the predicted height of the tree, in feet, at age 10 years?

- (A) 11.877
- (B) 11.889
- (C) 11.990

   (D) 12.341

## Answer C

Correct. Using a logarithmic regression,  $f(x) = 4.00397 + 3.46811 \ln x$ . The predicted height is f(10). (Note: Desmos requires using log mode to match the results from handheld graphing calculators.)

- 52. The life expectancy for a human in the United States can be modeled by the function L given by  $L(x) = 42.53 + 13.86 \ln x$ . L(x) gives the life expectancy, in years, for x decades after the year 1900. Based on the model, when is the first time that human life expectancy is predicted to be 80 years? (Note: 1 decade is 10 years).
  - (A) Late 1914
  - (B) Early 2003
  - (C) Late 2040s
  - (D) Early 2050s

## Answer C

Correct. Using the graphing calculator to solve L(x) = 80 gives x = 14.931. Therefore, the life expectancy is predicted to be 80 years around  $1900 + 14.93 \cdot 10 = 2049.3$ . This corresponds to the late 2040s.

- 53. If The Richter scale is a numerical scale that uses base 10 logarithms for measuring an earthquake's magnitude. The larger the number, the more intense the earthquake. As intensities increase multiplicatively by a factor of 10, the Richter scale increases additively by 1. Consider two earthquakes that occurred in the year 2022. An earthquake in Indonesia had a magnitude of 5.1, and an earthquake in Mexico had a magnitude of 2.5. Approximately how many times more intense was the Indonesia earthquake than the Mexico earthquake?
  - (A) 2.6
  - (B) 26
  - (C) 100
  - (D) 400

## **Answer D**

Correct. Because intensities increase multiplicatively by a factor of 10 as Richter scale values increase additively by 1, the earthquake is  $10^{(5.1-2.5)}$  times greater in intensity.

54.

III Year	Life Expectancy (years)
2000	76.75
2004	77.38
2008	78.19
2012	78.79
2016	78.86
2019	78.87

The table gives values of average life expectancy, in years, for a child born in a given year in the United States. A linear regression is used to construct a linear function model L, where t represents the birth year, t = 0 is the year 2000, and L(t) represents the life expectancy in years. For what year does the model predict that the life expectancy of a child born in that year will be 83 years?

(A) 2032	
(B) 2051	$\checkmark$
(C) 2056	
(D) 2086	

#### Answer **B**

Correct. This is the result of using a graphing calculator to determine the linear regression model, which is given as L = 0.1171t + 76.9883, where t is the number of years since 2000. Solving for t when L = 83 gives t = 51.328.

55. The electromagnetic force between two particular particles is related to the distance between the particles. This relationship is modeled by the function F, where  $F(d) = \frac{3.6}{d^2}$  for distance d, measured in centimeters, and force F(d), measured in Newtons. What is the average rate of change, in Newtons per centimeter, in the electromagnetic force if the distance between two particles is increased from 2.3 centimeters to 3.1 centimeters?

(A)	-0.382	~
(B)	-0.306	
(C)	0.375	
(D)	5.625	

#### Answer A

Correct. The average rate of change for a nonlinear function over a closed interval is found by dividing the change in force, F(3.1) - F(2.3), by the change in distance of 3.1 - 2.3 = 0.8 centimeters.

- 56. III An X-ray machine is used to eliminate germs in certain food processes. The intensity *I*, in millirads per hour, of X-rays produced by the machine is inversely proportional to the square of the distance *d*, in meters, from the machine. For a certain machine, the intensity is 26.5 millirads per hour at a distance of 4 meters. Based on this information and using the same machine, what is the intensity, in millirads per hour, at a distance of 3.3 meters?
  - (A) 18.037
  - (B) 29.176
  - (C) 32.121
  - (D) 38.935

#### Answer D

Correct. The rational function model,  $I = \frac{424}{d^2}$ , has been identified and accurately calculated from the given situation. The function has been evaluated correctly to draw a conclusion about the intensity of the X-ray machine at a distance of 3.3 meters.

The rate of people entering a subway car on a particular day is modeled by the function R, where  $R(t) = 0.03t^3 - 0.846t^2 + 6.587t + 1.428$  for  $0 \le t \le 20$ . R(t) is measured in people per hour, and t is measured in hours since the subway began service for the day.

- 57.  $\blacksquare$  Based on the model, at what value of t does the rate of people entering the subway car change from increasing to decreasing?
  - (A) t = 20
  - (B) t = 17.056
  - (C) t = 13.295
  - (D) t = 5.505

## **Answer D**

Correct. This is a relative maximum value, where R changes from increasing to decreasing.

58. For an arithmetic sequence  $S_n$ ,  $S_3 = 3$  and  $S_6 = 24$ . What is the value of  $S_{10} - S_8$ ?

(A) 2
(B) 7
(C) 14 ✓
(D) 288

## Answer C

Correct. The common difference of the arithmetic sequence is  $\frac{24-3}{6-3} = 7$ . The indices of the terms  $S_{10}$  and  $S_8$  are separated by 2, so the common difference is applied twice,  $7 \cdot 2$ .

- 59. When a certain car is initially purchased, its value is 20,000 dollars. If the car loses 9% of its value each year, when will the car's value be 10,000 dollars?
  - (A) Between the date of the initial purchase and 1 year
  - (B) Between 1 year and 2 years after its initial purchase
  - (C) Between 5 years and 6 years after its initial purchase
  - (D) Between 7 years and 8 years after its initial purchase

## Answer D

Correct. This is the result of using an exponential function  $20,000(1 - 0.09)^x = 10,000$  to appropriately model and solve the contextual scenario.

The function f is defined by  $f(x) = a \sin(b(x+c)) + d$ , for constants a, b, c, and d. In the xy-plane, the points (2,2) and (4,4) represent a minimum value and a maximum value, respectively, on the graph of f.

**60.** What are the values of a and d?

(A) $a = 1$ and $d = 3$	$\checkmark$
(B) $a=1$ and $d=2$	
(C) $a=2$ and $d=3$	
(D) $a=2$ and $d=2$	

## Answer A

Correct. The amplitude, a = 1, is half the distance from the maximum value to the minimum value (y -values).  $\frac{4-2}{2} = 1$ . The midline, y = 3, is the horizontal line that is halfway between the maximum value and the minimum value of the function. Therefore, d = 3.

61.

	f(x)
-2	10
-1	15
1	40
2	56

The table presents values for a function f at selected values of x. An exponential regression  $y = ab^x$  is used to model these data. What is the value of f(1.5) predicted by the exponential function model?

(A)	46.767	~
(B)	47.342	
(C)	47.800	
· `		

(D) 47.917

## Answer A

Correct. The exponential regression model for the data in the table is  $y = 24.076 \cdot (1.55681)^x$ . (Note: Use log mode in Desmos to achieve the same result as handheld graphing calculators.)

62. If The number of minutes of daylight per day for a certain city can be modeled by the function D given by  $D(t) = 160 \cos(\frac{2\pi}{365}(t-172)) + 729$ , where t is the day of the year for  $1 \le t \le 365$ . Which of the following best describes the behavior of D(t) on day 150?

(A) The number of minutes of daylight per day is increasing at a decreasing rate.

(B) The number of minutes of daylight per day is decreasing at a decreasing rate.

(C) The number of minutes of daylight per day is increasing at an increasing rate.

(D) The number of minutes of daylight per day is decreasing at an increasing rate.

## Answer A

Correct. By examining the graph of D on a graphing calculator, it can be determined that the point where t = 150 is above the point on the midline (80.75,729) and close to the relative maximum at (172,889). On the interval 80.75 < t < 172, the function is increasing in value. Because the graph of D is concave down on this interval, the rate of change is decreasing on the interval. Therefore, the number of minutes of daylight per day is increasing at a decreasing rate.

63. If The function f is defined by  $f(t) = 70 + 5 \sin(\frac{2\pi}{365}(t-90))$  for  $0 \le t \le 365$ . Function f models the temperature of water in a lake, in degrees Fahrenheit, t days since January 1. Based on the model, the predicted temperature of the water will be greater than 72 degrees Fahrenheit for approximately how many days?

(A)	113
()	

(B)	134
-----	-----

- (C) 231
- (D) 248

## Answer **B**

Correct. The graphing calculator is used to determine the distance between the *t*-coordinates of the two points of intersection of the graphs of  $f(t) = 70 + 5 \sin(\frac{2\pi}{365}(t-90))$  and y = 72 for  $0 \le t \le 365$ . Because the graph of *f* is above the line y = 72 between these two *t*-values, this gives the length of time, in days, 248.594 - 113.906, when the temperature is greater than 72.

64. If The function f is given by  $f(x) = 2\sin(4x) + \cos(2x)$ . Using the period of f, which of the following is the number of complete cycles of the graph of f in the xy-plane on the interval  $0 \le x \le 1000$ ?

(A)	159	

(B)	318
-----	-----

- (C) 602
- (D) 636

#### Answer **B**

Correct. Using the graphing calculator to examine the graph of f, the input-value interval between

consecutive maxima or consecutive minima is  $\pi$ . Therefore, the period is  $\pi$ . Calculating  $\frac{1000}{\pi} = 318.310$  results in 318 complete cycles on the interval  $0 \le x \le 1000$ .

65. In An open-top box is constructed by cutting squares that are x inches by x inches from the corners of an 11.2 -inch by 13.3-inch rectangular piece of cardboard, and then folding the sides of the box up to make walls. The volume of the box is modeled by the function V given by  $V(x) = 4x^3 - 49x^2 + 148.96x$ . Which of the following statements is true about the function V?

The contextual situation restricts the domain of V to 0 < x < 13.3, and the relative maximum value (A) of V on this domain is 2724.106. This is because the longer side of the cardboard determines the domain restriction.

The contextual situation restricts the domain of V to 0 < x < 11.2, and the relative maximum value (B) of V on this domain is 1141.504. This is because the shorter side of the cardboard determines the domain restriction.

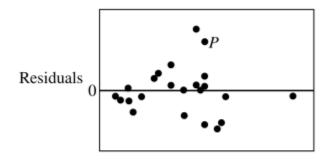
The contextual situation restricts the domain of V to 0 < x < 6.65, and the relative maximum value (C) of V on this domain is 133.929. This is because half of the longer side of the cardboard determines the domain restriction.

(D) The contextual situation restricts the domain of V to 0 < x < 5.6, and the relative maximum value of V on this domain is 133.929. This is because half of the shorter side of the cardboard determines the domain restriction.

#### Answer D

Correct. When a square of side x is cut from each corner of the piece of cardboard to form a box, x cannot be greater than half the length of the shorter side,  $\frac{11.2}{2} = 5.6$ . The maximum value within the restricted domain can be found to be 133.929 using the extrema feature on the graphing calculator.

66.



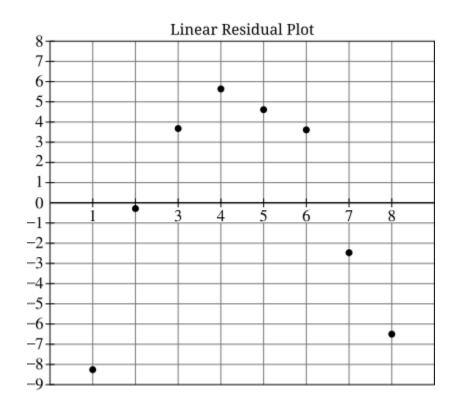
The students in a precalculus class measured each student's height and arm span, in centimeters. The students calculated a linear regression y = a + bx with heights as the input values and arm spans as the output values. The given residual plot has a point labeled P at coordinates (175,23.4). What does point P indicate in the context?

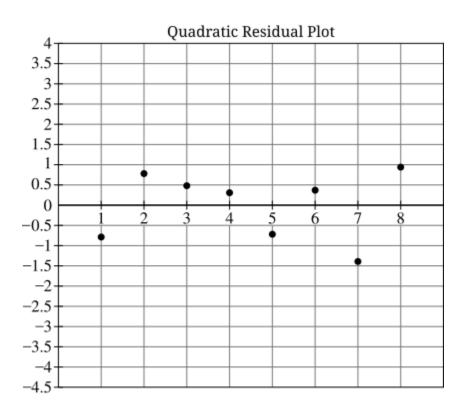
- (A) Because point P is above the x-axis, for the student with a height of 175 cm, the model overestimates the actual arm span value by 23.4 cm.
- (B) Because point P is above the x-axis, for the student with a height of 175 cm, the model underestimates the actual arm span value by 23.4 cm.
- (C) Because point P is above the x-axis, for the student with an arm span of 175 cm, the model overestimates the actual height value by 23.4 cm.
- (D) Because point P is above the x-axis, for the student with an arm span of 175 cm, the model underestimates the actual height value by 23.4 cm.

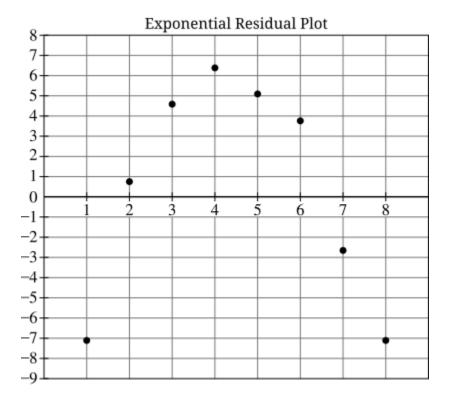
#### **Answer B**

Correct. Because input values are heights, the x-values indicate heights. The residual indicates that the actual arm span is 23.4 cm greater than the predicted value. Therefore, the predicted value is an underestimate.

67.







The graphs give three residual plots for linear, quadratic, and exponential regressions. Which of the following is true about the residual plots for these regressions?

- (A) The residual plots for the linear and exponential regressions are without pattern, and the residual plot for the quadratic regression shows a pattern. Therefore, the quadratic model is appropriate.
- (B) The residual plot for the quadratic regression is without pattern, and the residual plots for the linear and exponential regressions show patterns. Therefore, the linear and exponential models are appropriate.

The residual plots for the linear and exponential regressions are without pattern, and the residual plot

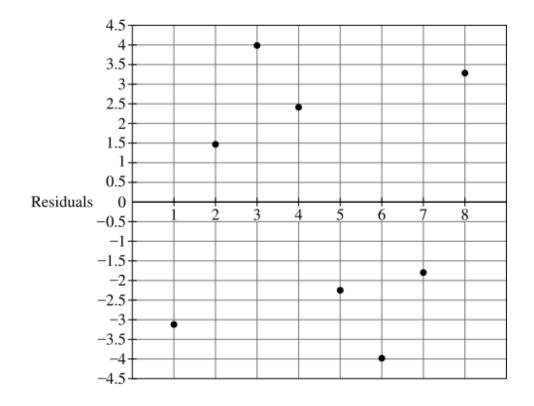
(C) for the quadratic regression shows a pattern. Therefore, the linear and exponential models are appropriate.

(D) The residual plot for the quadratic regression is without pattern, and the residual plots for the linear and exponential regressions show patterns. Therefore, the quadratic model is appropriate.

## Answer D

Correct. A model is appropriate if the residuals of the regression appear without pattern.

**68.** 



Students in a science class are constructing a model for a data set. The residual plot for their quadratic regression model is given. Which of the following is the best conclusion?

- (A) The quadratic model is appropriate because the residuals show a pattern.
- (B) The quadratic model is appropriate because the residuals show no pattern.
- (C) The quadratic model is not appropriate because the residuals show a pattern.
- (D) The quadratic model is not appropriate because the residuals show no pattern.

## Answer C

Correct. The residuals show a symmetric pattern that is roughly cubic. Because the residuals show a pattern, the model is not an appropriate choice.

69.

θ	0	π	$2\pi$
r=f( heta)	2	-4	3

The table gives all of the relative extrema of the polar function  $r = f(\theta)$  for all input values of  $\theta$  in the domain of the polar function. Which of the following statements is true about the graph of  $r = f(\theta)$  in the polar coordinate system?

- (A) The greatest distance between  $(f(\theta), \theta)$  and the origin is 3 because this is the maximum value of all the extrema.
- (B) The greatest distance between  $(f(\theta), \theta)$  and the origin is 3 because this is the maximum of the absolute values of all the extrema.
- (C) The greatest distance between  $(f(\theta), \theta)$  and the origin is 4 because this is the maximum value of all the extrema.
- (D) The greatest distance between  $(f(\theta), \theta)$  and the origin is 4 because this is the maximum of the absolute values of all the extrema.

#### Answer D

Correct. The extrema with the maximum absolute value is the extrema corresponding to a point that gives the greatest distance between the polar function and the origin.

- 70. The sinusoidal function y = g(x) has a period of  $\frac{5\pi}{2}$  and a minimum value at  $x = -\frac{3\pi}{2}$ . Which of the following statements with reason is true?
  - (A) The first maximum value for  $x > -\frac{3\pi}{2}$  occurs at  $x = -\frac{\pi}{2}$ , because the smallest interval of input values between the maximum and minimum output values is  $\pi$ .
  - (B) The first maximum value for  $x > -\frac{3\pi}{2}$  occurs at  $x = -\frac{\pi}{4}$ , because the smallest interval of input values between the maximum and minimum output values is  $\frac{1}{2}$  of the period of the sinusoidal function.
  - (C) The first maximum value for  $x > -\frac{3\pi}{2}$  occurs at  $x = \frac{\pi}{2}$ , because the smallest interval of input values between the maximum and minimum output values is  $2\pi$ .
  - (D) The first maximum value for  $x > -\frac{3\pi}{2}$  occurs at  $x = \pi$ , because the smallest interval of input values between the maximum and minimum output values is the period of the sinusoidal function.

## Answer B

Correct. Consecutive minima and maxima occur every interval of  $\frac{1}{2}$  of the period. If the period of g is  $\frac{5\pi}{2}$ , consecutive minima and maxima occur every  $\frac{1}{2} \cdot \frac{5\pi}{2} = \frac{5\pi}{4}$  units. Therefore,  $-\frac{3\pi}{2} + \frac{5\pi}{4} = -\frac{\pi}{4}$ .

- 71. The function g is given by  $g(x) = \sin x \cos x$  and has a period of  $2\pi$ . In order to define the inverse function of g, which of the following specifies a restricted domain for g and provides a rationale for why g is invertible on that domain?
  - (A)  $0 \le x \le \pi$ , because all possible values of g(x) occur without repeating on this interval.
  - (B)  $-\frac{\pi}{4} \le x \le \frac{3\pi}{4}$ , because all possible values of g(x) occur without repeating on this interval.
  - (C)  $0 \le x \le \pi$ , because the length of this interval is half of the period.
  - (D)  $-\frac{\pi}{4} \le x \le \frac{3\pi}{4}$ , because the length of this interval is half of the period.

### Answer B

Correct. Use the graphing calculator to examine the graph of  $g(x) = \sin x - \cos x$ . This is an interval of length  $\pi$ , and g is invertible on this interval because each output value of g is mapped from a unique input value. The minimum value of g, the maximum value of g, and all values of g in between those two values occur on the interval  $-\frac{\pi}{4} \le x \le \frac{3\pi}{4}$ .

x	g(x)
0	53
1	78
2	97
3	110
4	117

The table shows values for a function g at selected values of x.

- 72. Which of the following claim and explanation statements best fits these data?
  - (A) g is best modeled by a linear function, because the rate of change over consecutive equal-length inputvalue intervals is constant.
  - (B) g is best modeled by a linear function, because the change in the average rates of change over consecutive equal-length input-value intervals is constant.
  - (C) g is best modeled by a quadratic function, because the rate of change over consecutive equal-length input-value intervals is constant.
  - (D) g is best modeled by a quadratic function, because the change in the average rates of change over consecutive equal-length input-value intervals is constant.

### Answer D

Correct. The average rates of change over equal-length input-value intervals of length 1 for function g are 25, 19, 13, and 7. The change in these average rates of change, the 2nd differences, is a constant 6. Therefore, a quadratic function (degree 2) best models the data in the table.