FRQ_Review

x	1	2	3	4	5
$f(x)$	-10	-5	4	17	34

Let f be an increasing function defined for $x \geq 0$. The table gives values for $f(x)$ at selected values of x. The function g is given by $g(x) = \frac{x^3-14x-27}{x+2}$.

1. **Part A**

(i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(5)$ as a decimal approximation, or indicate that it is not defined. Show the work that leads to your answer.

(ii) Find the value of $f^{-1}(4)$, or indicate that it is not defined.

Part A

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2

The student response includes both of these criteria.

- Value of $h(5)$ with supporting work
- Value of $f^{-1}(4)$

Model Solution

(i) $h(5) = g(f(5)) = g(34) = \frac{(34)^3-14(34)-27}{34+2} = 1077.806$

(ii) Because f is increasing on its domain, f^{-1} exists. From the table, $f^{-1}(4) = 3$.

2. **Part B**

(i) Find all values of x, as decimal approximations, for which $g(x) = 3$, or indicate there are no such values.

(ii) Determine the end behavior of g as x decreases without bound. Express your answer using the mathematical notation of a limit.

Part B

FRQ_Review

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2

The student response includes both of these criteria.

- Answer of $x = 4.875$
- End behavior with limit notation

Model Solution

(i) $g(x) = 3 \rightarrow \frac{x^3-14x-27}{x+2} = 3$

$x = 4.875$

(ii) As x decreases without bound, eventually $g(x)$ increases without bound. Therefore, $\lim_{x \to -\infty} g(x) = \infty$.

3. Part C

(i) Based on the table, which of the following function types best models function f: linear, quadratic, exponential, or logarithmic?

(ii) Give a reason for your answer in part C(i) based on the relationship between the change in the output values of f and the change in the input values of f. Refer to the values in the table in your reasoning.

Part C

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2

The student response includes both of these criteria.

- Answer of quadratic function
- Reason for quadratic function (Note: reference to a quadratic regression is not a sufficient reason.)

Model Solution

(i) f is best modeled by a quadratic function.

(ii)

FRQ_Review

x	1st Differences in $f(x)$-values	2nd Differences in $f(x)$-values
1		
2	$-5-(-10)=5$	
3	$4-(-5)=9$	$9-5=4$
4	$17-4=13$	$13-9=4$
5	$34-17=17$	$17-13=4$

Because the 2nd differences in the output values are a constant 4 over consecutive equal-length input-value intervals, a quadratic model is best.

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x	1	2	3	4	5
$f(x)$	1	3	5	3	1

The domain of f consists of the five real numbers $1, 2, 3, 4$, and 5. The table defines the function f for these values. The function g is given by $g(x) = 2\ln x$.

4.

Part A

(i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(4)$ as a decimal approximation, or indicate that it is not defined. Show the work that leads to your answer.

(ii) Find all values of x for which $f(x) = 3$, or indicate that there are no such values.

Part C

(i) Does the function f have an inverse function?

(ii) Give a reason for your answer in part C(i) based on properties of the function f. Refer to the values in the table in your reasoning.

Part A

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2

FRQ_Review

The student response includes both of these criteria.

- Value of $h(4)$ with supporting work
- Values for which $f(x) = 3$

Model Solution

(i) $h(4) = g(f(4)) = g(3) = 2\ln 3 = 2.197$

(ii) From the table, $f(x) = 3$ when $x = 2$ and $x = 4$.

Part B

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2

The student response includes both of these criteria.

- Answer of $x = 4.482$ (OR 4.481)
- End behavior with limit notation

Model Solution

(i) $g(x) = 3 \Rightarrow 2\ln x = 3$

$x = 4.482$ (OR 4.481)

(ii) The function g is increasing. As x increases without bound, $g(x)$ increases without bound. Therefore, $\lim_{x\to\infty} g(x) = \infty$.

Part C

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2

The student response includes both of these criteria.

- Answer of f does not have an inverse function
- Reason for no inverse function (Note: reference to “fails the horizontal line test” is not a sufficient reason.)

FRQ_Review

Model Solution

(i) f does not have an inverse function on its domain of the five real numbers $1, 2, 3, 4$, and 5.

(ii) There are output values of f that are not mapped from unique input values. (The function f is not one-to-one.) Because $f(1) = 1$ and $f(5) = 1$, the inverse function on this domain does not exist.

A reason that only states "fails the horizontal line test" is not sufficient.

An ecologist began studying a certain type of plant species in a wetlands area in 2013. In 2015 ($t = 2$), there were 59 plants. In 2021 ($t = 8$), there were 118 plants.

The number of plants of this species can be modeled by the function P given by $P(t) = ab^t$, where $P(t)$ is the number of plants during year t, and t is the number of years since 2013

5. **Part A**

(i) Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $P(t)$.

(ii) Find the values for a and b as decimal approximations.

Part C

In which t-value, $t = 6$ years or $t = 20$ years, should the ecologist have more confidence when using the model P ? Give a reason for your answer in the context of the problem.

Part A

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2

The student response includes both of these criteria.

- Two equations
- Values of a and b

Model Solution

(i) Because $P(2) = 59$ and $P(8) = 118$, two equations to find a and b are

FRQ_Review

$ab^2 = 59$

$ab^8 = 118.$

(ii)

$a = \frac{59}{b^2} \Rightarrow \left(\frac{59}{b^2}\right)b^8 = 118$

$b = \left(\frac{118}{59}\right)^{1/6} = 1.122462$

$a = 46.828331$

$P(t) = 46.828(1.122)^t$

Part B

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2	3

The student response includes all three of these criteria.

- Correct average rate of change based on exponential $P(t)$ from Part A
- Correct estimate for $t = 10$ based on average rate of change found in (i)
- Correct answer with explanation

Model Solution

(i) $\frac{P(8)-P(2)}{8-2} = \frac{(118-59)}{6} = 9.833327$

The average rate of change is 9.833 plants per year.

(ii) The average rate of change is $r = \frac{P(8)-P(2)}{8-2} = 9.833327$.

The secant line between point $(2, P(2))$ and point $(8, P(8))$ is given by $y = y_1 + \left(\frac{P(8)-P(2)}{8-2}\right)(x - x_1)$, where $(x_1,\ y_1)$ can be either one of the points.

Estimates using the average rate of change are given by

$y = P(2) + r(x-2)$

OR

$y = P(8) + r(x-8)$.
Both of these produce the same estimate.

FRQ_Review

For $x = 10$,

$y = 59 + r(10 - 2) = 137.667$.

The number of plants for $t = 10$ years was approximately 137 or 138.

(iii) The estimate using the average rate of change is the y-coordinate of a point on the secant line that passes through $(2, P(2))$ and $(8, P(8))$. Because the graph of P is concave up on the interval $(-\infty, \infty)$, the secant line is below the graph of P outside of the interval $(2, 8)$.

Therefore, the estimate using the average rate of change is less than the value of $P(t)$ for $t > 10$.

Part C

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1

The student response includes this criterion.

- Answer with reason based on use of the displayed exponential $P(t)$ from Part A

Model Solution

The ecologist should have more confidence in using the model for $t = 6$ years. There is insufficient information to know how many years the exponential model can be extended above the maximum time provided in the data ($t = 8$) to make reasonable predictions. On the other hand, it is appropriate to use the regression model to estimate values at times that fall between the minimum time ($t = 2$) and the maximum time ($t = 8$) provided in the data.

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6. **Part B**

(i) Use the given data to find the average rate of change of the scores, in points per month, from $t = 0$ to $t = 3$ months. Express your answer as a decimal approximation. Show the computations that lead to your answer.

(ii) Interpret the meaning of your answer from part B (i) in the context of the problem.

(iii) Consider the average rates of change of R from $t = 3$ to $t = p$ months, where $p > 3$. Are these average rates of change less than or greater than the average rate of change from $t = 0$ to $t = 3$ months found in part B (i)? Explain your reasoning. Your explanation should include a reference to the graph of R.

Part B

FRQ_Review

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2	3

The student response includes all three of these criteria.

- Correct average rate of change based on logarithmic $R(t)$ from Part A
- Interpretation provided includes the idea that there is a loss of points per month at the correct rate
- Correct answer with explanation

Model Solution

(i) $\frac{R(3)-R(0)}{3-0} = -1.387$

The average rate of change is -1.387 points per month.

(ii) From $t = 0$ to $t = 3$ months, on average, the group's score was decreasing at a rate of 1.387 points per month.

(iii) The average rates of change are found using secant lines that pass through the points $(3, R(3))$ and $(p, R(p))$, where $p > 3$.

Because R is logarithmic and decreasing, its graph is concave up. These secant lines are above the graph of R, and the slopes of the lines are increasing as t increases.

Therefore, the average rates of change of R, in points per month, from $t = 3$ to $t = p$ months are greater than the average rate of change from $t = 0$ to $t = 3$ months.

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Directions:

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

FRQ_Review

7.

Part A

The functions g and h are given by

$$g(x) = \log_4(2x)$$

$$h(x) = \frac{(e^x)^5}{e^{(1/4)}}.$$

(i) Solve $g(x) = 3$ for values of x in the domain of g.

(ii) Solve $h(x) = e^{(1/2)}$ for values of x in the domain of h.

Part B

The functions j and k are given by

$$j(x) = \log_{10}(x+1) - 5\log_{10}(2-x) + \log_{10} 3$$

$$k(x) = \sec x - \cos x.$$

(i) Rewrite $j(x)$ as a single logarithm base 10 without negative exponents in any part of the expression. Your result should be of the form $\log_{10}(\text{expression})$.

(ii) Rewrite $k(x)$ as a product involving $\tan x$ and $\sin x$ and no other trigonometric functions.

Part C

The function m is given by

$$m(x) = 2\tan^{-1}\left(\sqrt{3}\pi x\right).$$

Find all values in the domain of m that yield an output value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

Part A

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2

The student response includes both of these criteria.

FRQ_Review

- Solution to $g(x) = 3$
- Solution to $h(x) = e^{1/2}$

Model Solution

(i)

$g(x) = 3$

$\log_4(2x) = 3$

$4^3 = 2x$

$x = \frac{4^3}{2} = 32$

(ii)

$h(x) = e^{1/2}$

$\frac{(e^x)^5}{e^{1/4}} = e^{1/2}$

$e^{(5x-1/4)} = e^{1/2}$

$5x - \frac{1}{4} = \frac{1}{2}$

$5x = \frac{3}{4}$

$x = \frac{3}{20}$

Part B

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2

The student response includes both of these criteria.

- Expression for $j(x)$
- Expression for $k(x)$

Model Solution

(i) $j(x) = \log_{10}(x+1) - 5\log_{10}(2-x) + \log_{10}3$

$j(x) = \log_{10}(x+1) - \log_{10}(2-x)^5 + \log_{10}3$

FRQ_Review

$j(x) = \log_{10}\left(\frac{3(x+1)}{(2-x)^5}\right), -1 < x < 2$

(ii) $k(x) = \sec\ x - \cos\ x$

$k(x) = \frac{1}{\cos\ x} - \cos\ x$

$k(x) = \frac{1-\cos^2 x}{\cos\ x}$

$k(x) = \frac{\sin^2 x}{\cos\ x} = \tan\ x \sin\ x, \cos\ x \neq 0$

Part C

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2 ✓

The student response includes both of these criteria.

- $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$
- Input value

Model Solution

$m(x) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \Rightarrow 2\tan^{-1}\left(\sqrt{3}\pi x\right) = \frac{\pi}{3}$

$\tan^{-1}(\sqrt{3}\pi x) = \frac{\pi}{6}$

$\sqrt{3}\pi x = \tan\left(\frac{\pi}{6}\right)$

$\sqrt{3}\pi x = \frac{1}{\sqrt{3}}$

$x = \frac{1}{3\pi}$

FRQ_Review

Directions:

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos(\frac{\pi}{2})$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

8. Part A

The functions g and h are given by

$$g(x) = 3\ln x - \frac{1}{2}\ln x$$

$$h(x) = \frac{\sin^2 x - 1}{\cos x}.$$

(i) Rewrite $g(x)$ as a single natural logarithm without negative exponents in any part of the expression. Your result should be of the form $\ln(\text{expression})$.

(ii) Rewrite $h(x)$ as an expression in which $\cos x$ appears once and no other trigonometric functions are involved.

Part A

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2

The student response includes both of these criteria.

- Expression for $g(x)$
- Expression for $h(x)$

Model Solution

(i) $g(x) = 3\ln x - \frac{1}{2}\ln x$

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$g(x) = \frac{5}{2}\ln\, x$

$g(x) = \ln(x^{5/2}),\, x > 0$

–OR–

$g(x) = 3\ln\, x - \frac{1}{2}\ln\, x$

$g(x) = \ln(x^3) - \ln(x^{1/2})$

$g(x) = \ln(\frac{x^3}{x^{1/2}})$

$g(x) = \ln(x^{5/2}),\, x > 0$

(ii) $h(x) = \frac{\sin^2 x - 1}{\cos\, x}$

$h(x) = \frac{-\cos^2 x}{\cos\, x}$

$h(x) = -\cos\, x,\, \cos\, x \neq 0$

9. Part B

The functions j and k are given by

$j(x) = 2(\sin x)(\cos x) - \cos\, x$

$k(x) = 8e^{(3x)} - e.$

(i) Solve $j(x) = 0$ for values of x in the interval $\left[0, \frac{\pi}{2}\right]$.

(ii) Solve $k(x) = 3e$ for values of x in the domain of k.

Part B

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2 ✓

The student response includes both of these criteria.

- Solutions to $j(x) = 0$
- Solution to $k(x) = 3e$

Model Solution

FRQ_Review

(i) $2(\sin x)(\cos x) - \cos x = 0$

$\cos\ x(2\sin\ x - 1) = 0$

$\cos\ x = 0$ or $2\sin\ x - 1 = 0$

$\cos\ x = 0$ or $\sin\ x = \frac{1}{2}$

$x = \frac{\pi}{2}$ or $x = \frac{\pi}{6}$

(ii) $8e^{(3x)} - e = 3e$

$8e^{(3x)} = 4e$

$e^{(3x)} = \frac{e}{2}$

$\ln(e^{(3x)}) = \ln(\frac{e}{2})$

$3x = \ln(\frac{e}{2})$

$x = \frac{1}{3}\ln(\frac{e}{2})$

$x = \frac{\ln\ e - \ln\ 2}{3}$

$x = \frac{1 - \ln\ 2}{3}$

10. Part C

The function m is given by

$$m(x) = \cos(2x) + 4.$$

Find all input values in the domain of m that yield an output value of $\frac{9}{2}$.

Part C

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	2 ✓

The student response includes both of these criteria.

- Values of θ in $0 \leq \theta \leq 2\pi$ with $\cos\ \theta = \frac{1}{2}$
- General solution expression

FRQ_Review

Model Solution

$m(x) = \frac{9}{2} \Rightarrow \cos(2x) + 4 = \frac{9}{2}$

$\cos(2x) = \frac{1}{2}$

$2x = \frac{\pi}{3} + 2\pi n$ or $2x = \frac{5\pi}{3} + 2\pi n$

$x = \frac{\pi}{6} + \pi n$ or $x = \frac{5\pi}{6} + \pi n$, where n is any integer.